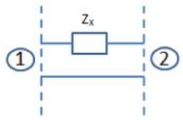
scattering matrix calculation



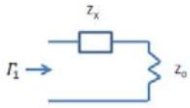
property

$S_{11} = S_{22}$ (symmetry)

$S_{21} = S_{12}$ (reciprocity)

Series impedance

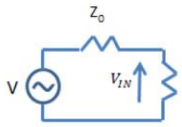
$$S_{11} = \Gamma_1\Big|_{\substack{a_2=0 \\ Z_L=Z_0}}$$



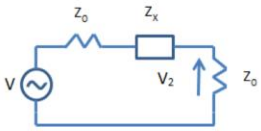
$$S_{11} = \Gamma_1 = \frac{(Z_X+Z_0)-Z_0}{(Z_X+Z_0)+Z_0} = \frac{Z_X}{Z_X+2Z_0}$$

$$S_{21} = \frac{b_2}{a_1}\Big|_{\substack{a_2=0 \\ Z_L=Z_0}} = \frac{V_2}{V_{IN}}$$

where $a_1 = V_{IN}$ (incident wave)



$$V_{IN} = \frac{V}{2}$$



$$b_2 = V_2 = V \cdot \frac{Z_0}{2Z_0 + Z_X}$$

$$S_{21} = V \cdot \frac{Z_0}{2Z_0 + Z_X} \cdot \frac{2}{V} = \frac{2Z_0}{2Z_0 + Z_X}$$

raised structure property

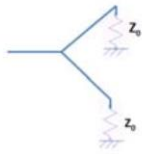
$$S_{11} + S_{12} = 1$$

T network

property

$$S_{11} = S_{22} = S_{33} \quad \text{symmetry}$$
$$S_{12} = S_{23} = S_{31} \quad \text{symmetry}$$
$$S_{13} = S_{32} = S_{21}$$

$$S_{11} = \Gamma_1\Big|_{\substack{a_2=a_3=0 \\ Z_2=Z_3=Z_0}}$$

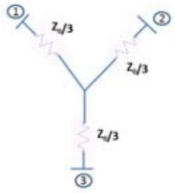


$$\Gamma_1 = \frac{\frac{Z_0}{2}-Z_0}{\frac{Z_0}{2}+Z_0} = \frac{Z_0-2Z_0}{Z_0+2Z_0} = -\frac{1}{3}$$

Being a raised structure

$$S_{11} + S_{12} + S_{13} = 1 \Rightarrow S_{13} = S_{12} = \frac{2}{3}$$

Resistive power divider



$$Z_{IN} = Z_1\Big|_{\substack{a_2=a_3=0 \\ Z_2=Z_3=Z_0}} = \frac{\left(\frac{Z_0}{3}+Z_0\right)}{2} + \frac{Z_0}{3} = Z_0$$

then $S_{11} = 0$

Being a raised structure

$$S_{11} + S_{12} + S_{13} = 1 \Rightarrow S_{12} = S_{13} = \frac{1}{2}$$