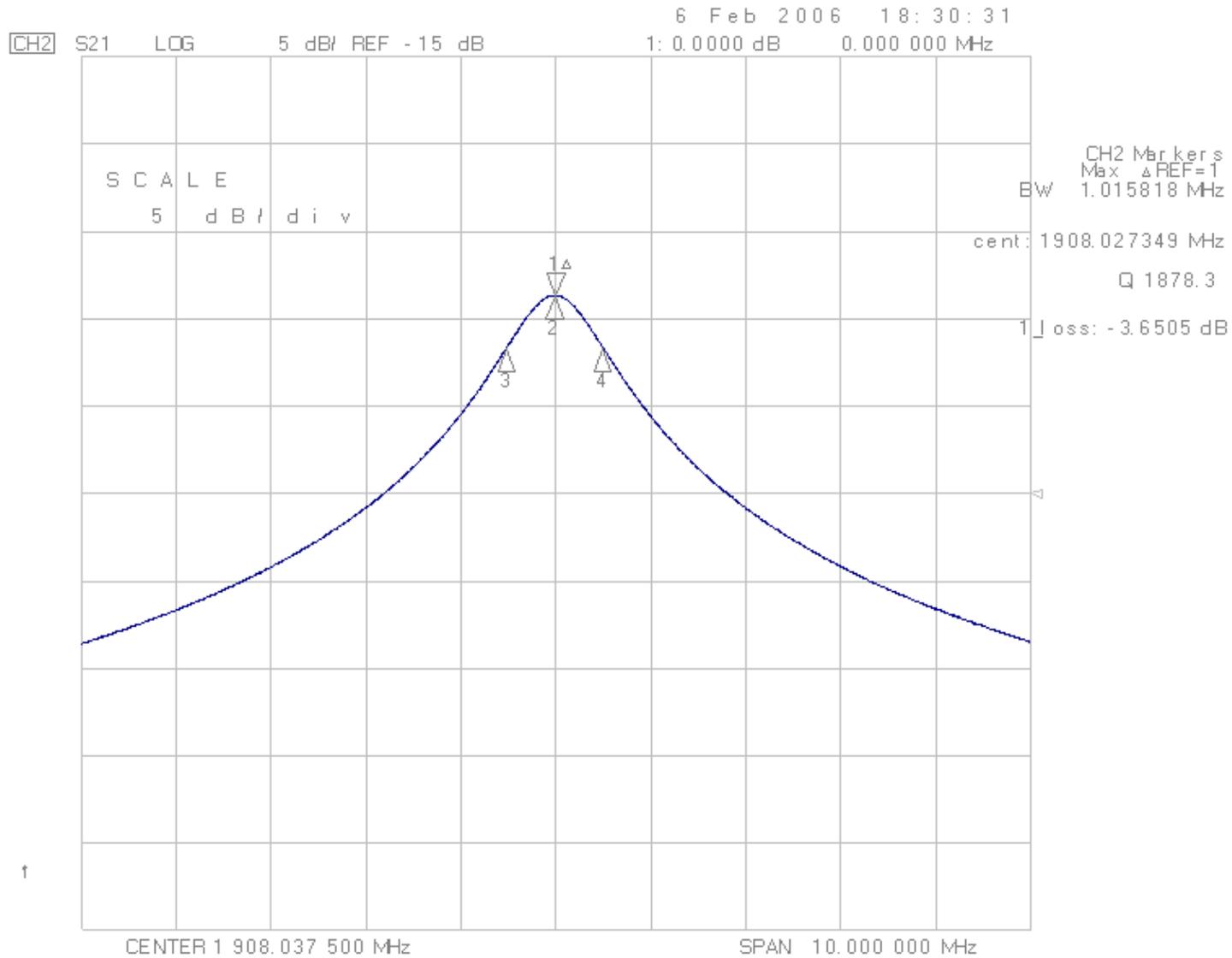


# Cavity transmission measurement



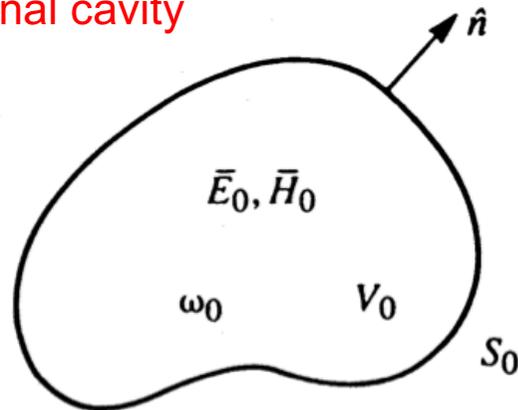
# **Slater theorem and applications**

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# Cavity shape perturbation

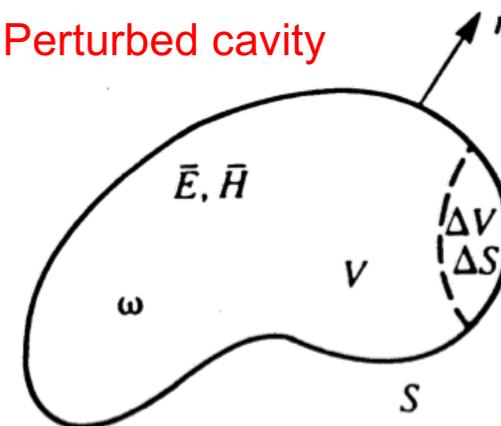
Inserting small metallic objects into a cavity or slightly deforming the shape of the cavity can be treated by the perturbation technique (**Slater theorem**)

Original cavity



$$\begin{aligned}\nabla \times \mathbf{E}_0 &= -j\omega_0\mu\mathbf{H}_0 \\ \nabla \times \mathbf{H}_0 &= j\omega_0\varepsilon\mathbf{E}_0\end{aligned}$$

Perturbed cavity



$$\begin{aligned}\nabla \times \mathbf{E} &= -j\omega\mu\mathbf{H} \\ \nabla \times \mathbf{H} &= j\omega\varepsilon\mathbf{E}\end{aligned}$$

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}_0^*) - \mathbf{E}_0^* \cdot (\nabla \times \mathbf{H}) = \nabla \cdot (\mathbf{E}_0^* \times \mathbf{H}) = j\omega_0\mu\mathbf{H}_0 \cdot \mathbf{H}_0^* - j\omega\varepsilon\mathbf{E}_0^* \cdot \mathbf{E}$$

$$\mathbf{H}_0^* \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H}_0^*) = \nabla \cdot (\mathbf{E} \times \mathbf{H}_0^*) = -j\omega\mu\mathbf{H}_0^* \cdot \mathbf{H} + j\omega_0\varepsilon\mathbf{E} \cdot \mathbf{E}_0^*$$

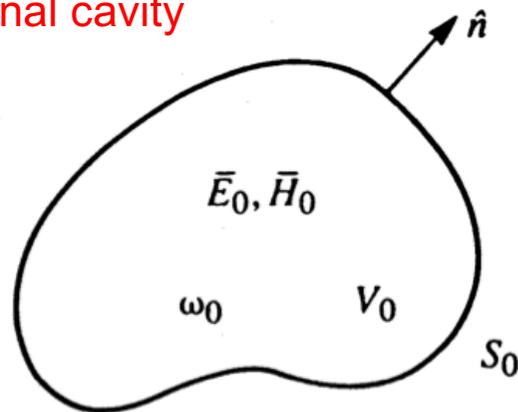
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$$\begin{aligned} & \int_V \nabla \cdot (\mathbf{E} \times \mathbf{H}_0^* + \mathbf{E}_0^* \times \mathbf{H}) dV = \oint_S (\mathbf{E} \times \mathbf{H}_0^* + \mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S} = \\ \mathbf{n} \times \mathbf{E} = 0 & \text{ on } S \quad \rightarrow \quad = \oint_S (\mathbf{E}_0^* \times \mathbf{H}) d\mathbf{S} = -j(\omega - \omega_0) \int_V (\varepsilon\mathbf{E} \cdot \mathbf{E}_0^* + \mu\mathbf{H}_0^* \cdot \mathbf{H}) dV \end{aligned}$$

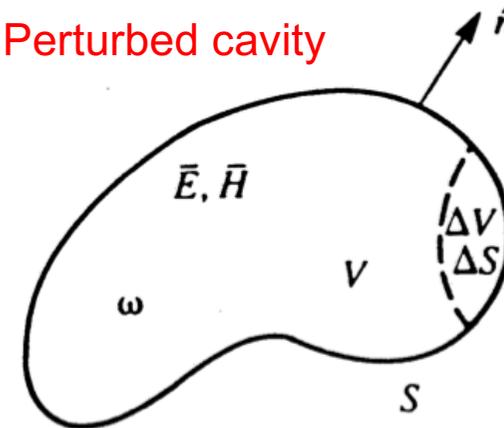
# Cavity shape perturbation

Inserting small metallic objects into a cavity or slightly deforming the shape of the cavity can be treated by the perturbation technique (**Slater theorem**)

Original cavity



Perturbed cavity



$$\int_V \nabla \cdot (\mathbf{E} \times \mathbf{H}_0^* + \mathbf{E}_0^* \times \mathbf{H}) dV = \oint_S (\mathbf{E} \times \mathbf{H}_0^* + \mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S} =$$

$$= \oint_S (\mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S} = -j(\omega - \omega_0) \int_V (\epsilon \mathbf{E} \cdot \mathbf{E}_0^* + \mu \mathbf{H}_0^* \cdot \mathbf{H}) dV$$

$$S = S_0 - \Delta S \quad \oint_{S_0} (\mathbf{E}_0^* \times \mathbf{H}) d\mathbf{S} - \oint_{\Delta S} (\mathbf{E}_0^* \times \mathbf{H}) d\mathbf{S} = \oint_{\Delta S} (\mathbf{E}_0^* \times \mathbf{H}) d\mathbf{S}$$

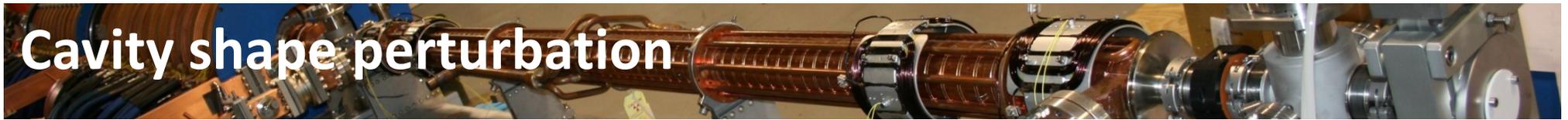
$$\uparrow$$

$$\mathbf{n} \times \mathbf{E}_0 = 0 \quad \text{on } S_0$$

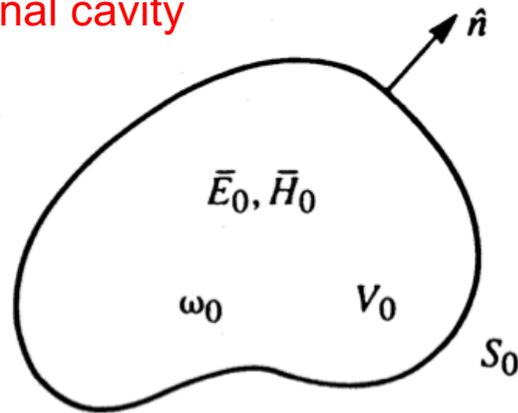
$$\omega - \omega_0 = -j \frac{\oint_{\Delta S} (\mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S}}{\int_V (\epsilon \mathbf{E} \cdot \mathbf{E}_0^* + \mu \mathbf{H}_0^* \cdot \mathbf{H}) dV}$$

Exact (but not very useful) expression since E, H in the perturbed cavity are unknown.

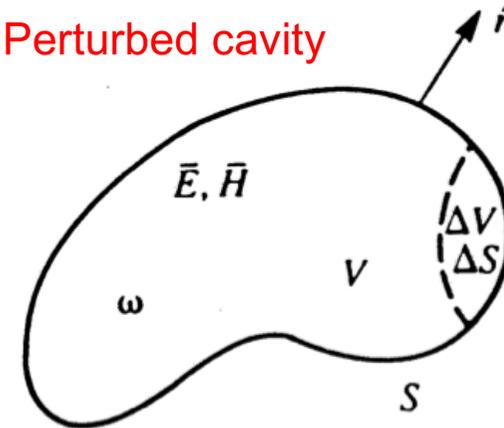
# Cavity shape perturbation



Original cavity



Perturbed cavity



$$\mathbf{E} \approx \mathbf{E}_0$$

$$\mathbf{H} \approx \mathbf{H}_0$$

small perturbation

$$\oint_{\Delta S} (\mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S} \approx \oint_{\Delta S} (\mathbf{E}_0^* \times \mathbf{H}_0) \cdot d\mathbf{S} = -j\omega_0 \int_{\Delta V} (\epsilon |\mathbf{E}_0|^2 - \mu |\mathbf{H}_0|^2) dV$$

Poyting Theor.

$$\omega - \omega_0 = -j \frac{\oint_{\Delta S} (\mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S}}{\int_V (\epsilon \mathbf{E} \cdot \mathbf{E}_0^* + \mu \mathbf{H}_0^* \cdot \mathbf{H}) dV}$$

It is essentially the energy stored in the cavity and it will not change much with the perturbation.

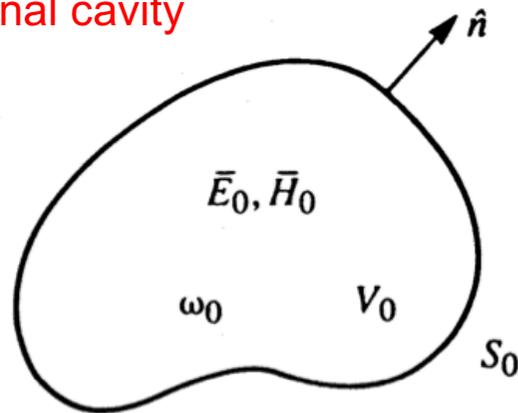
**Change in stored electric/magnetic energy**

$$\frac{\omega - \omega_0}{\omega_0} \approx \frac{\int_{\Delta V} (\mu |\mathbf{H}_0|^2 - \epsilon |\mathbf{E}_0|^2) dV}{\int_{V_0} (\epsilon |\mathbf{E}_0|^2 + \mu |\mathbf{H}_0|^2) dV} = \frac{\Delta W_m - \Delta W_e}{W_m + W_e}$$

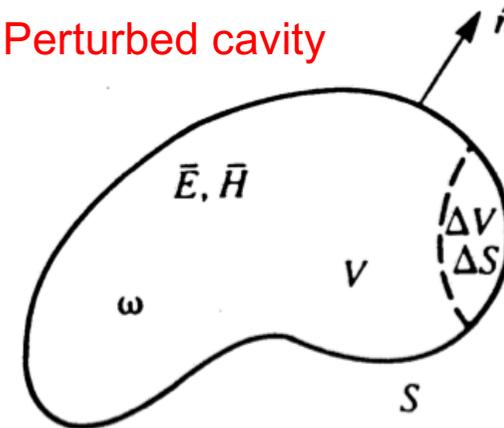
**Total energy stored**

# Applications: tuning of a cavity

Original cavity



Perturbed cavity



$$\mathbf{E} \approx \mathbf{E}_0$$

$$\mathbf{H} \approx \mathbf{H}_0$$

small perturbation

$$\frac{\Delta\omega}{\omega_0} \approx \frac{\int_{\Delta V} (\mu|\mathbf{H}_0|^2 - \varepsilon|\mathbf{E}_0|^2) dV}{\int_{V_0} (\varepsilon|\mathbf{E}_0|^2 + \mu|\mathbf{H}_0|^2) dV} = \frac{1}{W_{tot}} \int_{\Delta V} \mu|\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}} \int_{\Delta V} \varepsilon|\mathbf{E}_0|^2 dV$$

The frequency shift depends on the kind and the amplitude of the unperturbed cavity field.

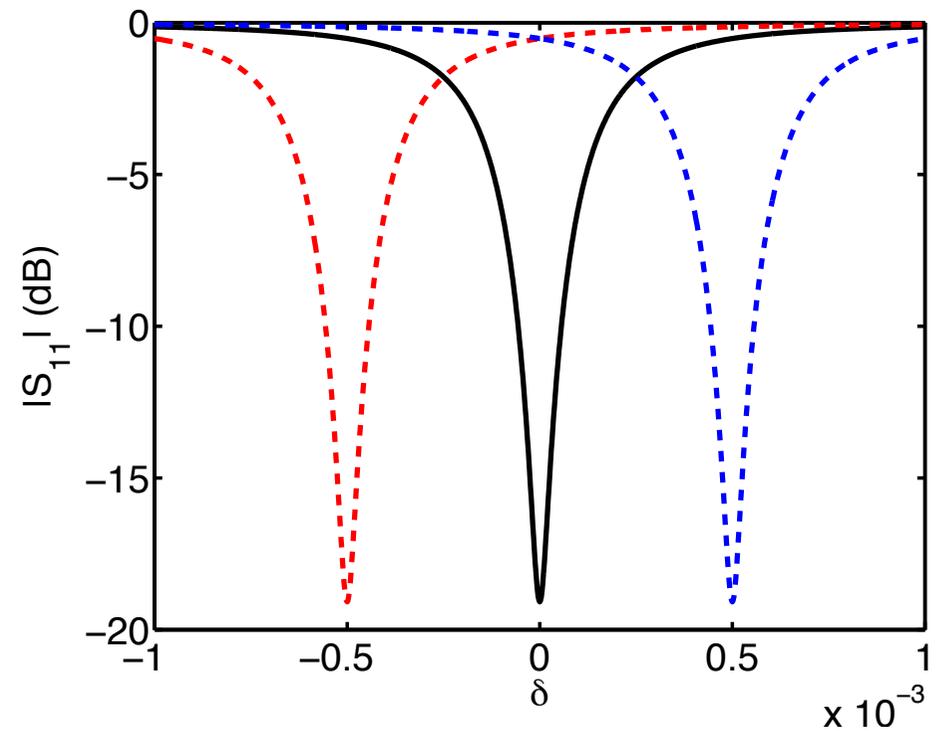
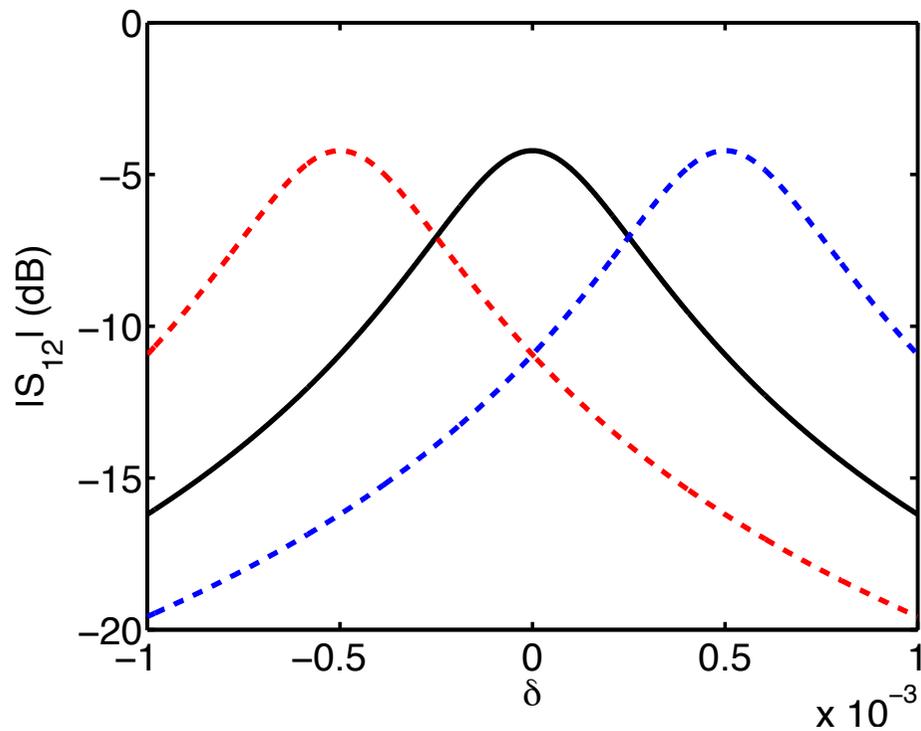
# Applications: tuning of a cavity (S parameters)

$$\frac{\Delta\omega}{\omega_0} \approx \frac{1}{W_{tot}} \int_{\Delta V} \mu |\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}} \int_{\Delta V} \epsilon |\mathbf{E}_0|^2 dV$$

Unperturbed

Electric Field

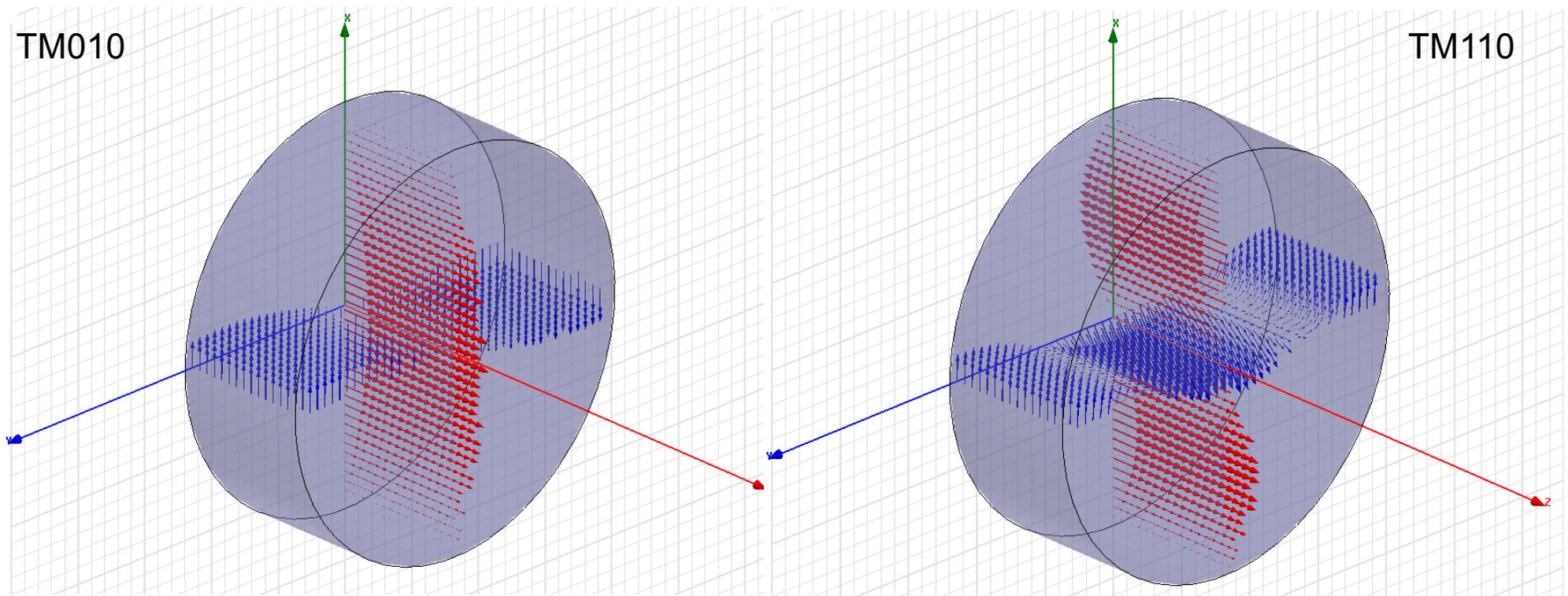
Magnetic Field



# Applications: tuning of different cavity modes

The same tuners affect different modes in different ways ...

$$\frac{\Delta\omega}{\omega_0} \approx \frac{1}{W_{tot}} \int_{\Delta V} \mu |\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}} \int_{\Delta V} \varepsilon |\mathbf{E}_0|^2 dV$$



**Task:** for a given cavity with two tuners, you can tune the resonant frequency two modes simultaneously.

Courtesy of L. Ficcadenti

Andrea.Mostacci@uniroma1.it

# Applications: bead pull measurement

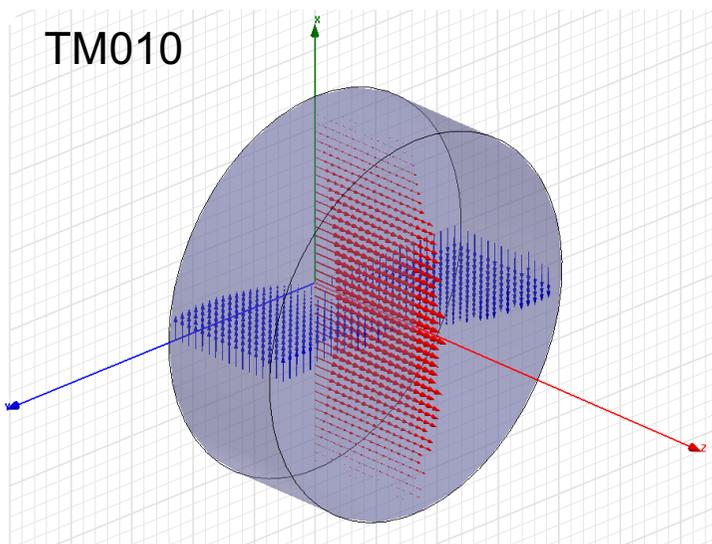
Introducing a small metallic object into the interior of the cavity should perturb the frequency in a similar way by an amount depending upon the local fields, and thus we could use the **frequency shift to measure the field strength** at an interior point.

$$\frac{\Delta\omega}{\omega_0} \approx \frac{1}{W_{tot}} \int_{\Delta V} \mu |\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}} \int_{\Delta V} \varepsilon |\mathbf{E}_0|^2 dV$$

## Measurements

$$\frac{\Delta\omega}{\omega_0} \approx \left( k_{\parallel}^H \mu \frac{|H_z|^2}{W_{tot}} + k_{\perp}^H \mu \frac{|\mathbf{H}_{\perp}|^2}{W_{tot}} \right) - \left( k_{\parallel}^E \varepsilon \frac{|E_z|^2}{W_{tot}} + k_{\perp}^E \varepsilon \frac{|\mathbf{E}_{\perp}|^2}{W_{tot}} \right)$$

Theory and/or calibration in known cavities



Accelerating field on the cavity axis

$$\frac{|E_z|^2}{W_{tot}} \approx -\frac{1}{k_{\perp}^E \varepsilon} \frac{\Delta\omega}{\omega_0}$$

## Measurements

# Applications: bead pull measurement

Introducing a small metallic object into the interior of the cavity should perturb the frequency in a similar way by an amount depending upon the local fields, and thus we could use the **frequency shift to measure the field strength** at an interior point.

$$\frac{\Delta\omega}{\omega_0} \approx \frac{1}{W_{tot}} \int_{\Delta V} \mu |\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}} \int_{\Delta V} \varepsilon |\mathbf{E}_0|^2 dV$$

## Measurements

$$\frac{\Delta\omega}{\omega_0} \approx \left( k_{\parallel}^H \mu \frac{|H_z|^2}{W_{tot}} + k_{\perp}^H \mu \frac{|\mathbf{H}_{\perp}|^2}{W_{tot}} \right) - \left( k_{\parallel}^E \varepsilon \frac{|E_z|^2}{W_{tot}} + k_{\perp}^E \varepsilon \frac{|\mathbf{E}_{\perp}|^2}{W_{tot}} \right)$$

**Theory and/or calibration in known cavities**

## In general

Metal objects affect E and H field

Dielectric objects affect E field



It is possible to measure H field with two measurements

# Fast automatic field measurement: the idea

$$\frac{|E_z|^2}{W_{tot}} \approx -\frac{1}{k_{\perp}^E \varepsilon} \frac{\Delta\omega}{\omega_0}$$

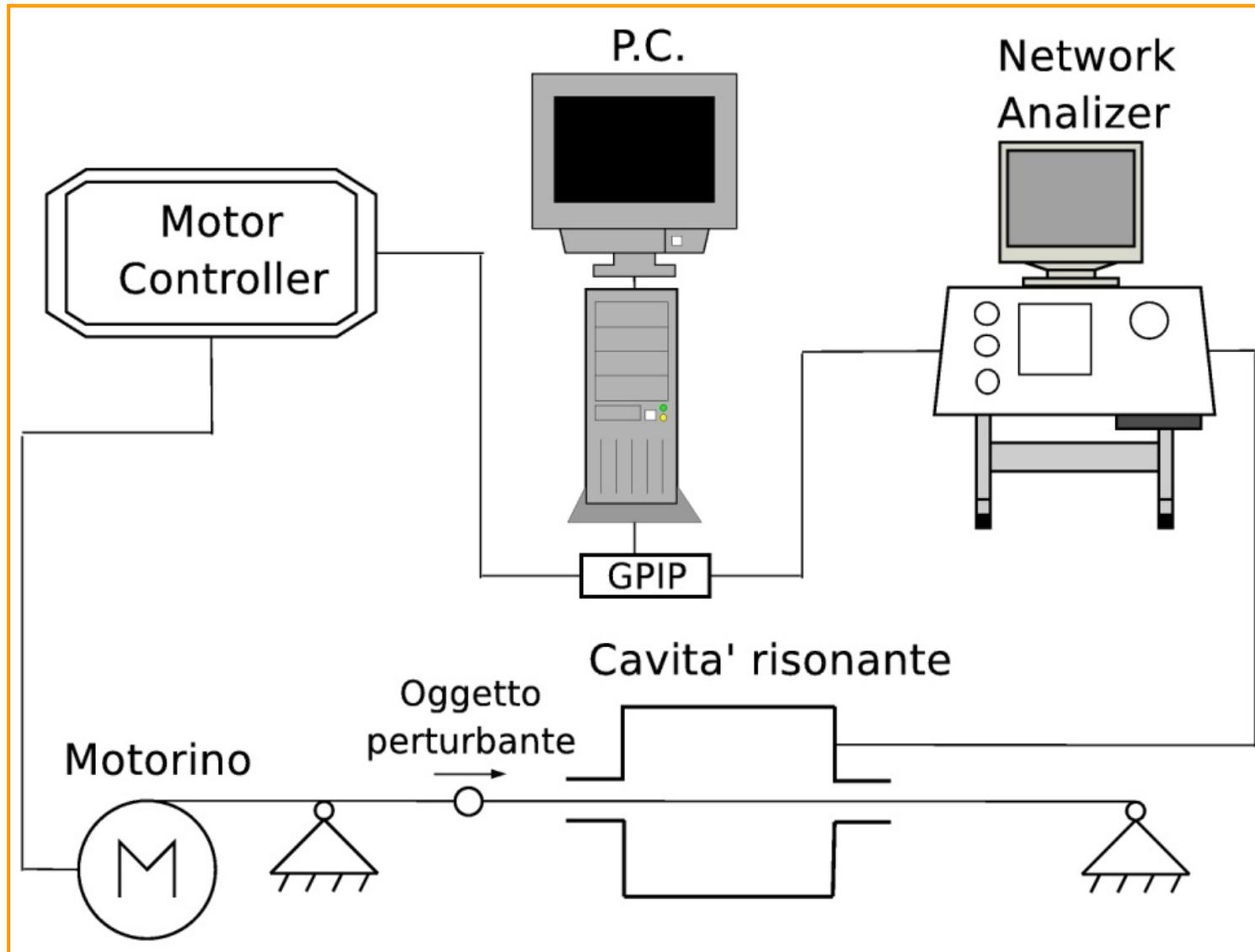
Measurements

$$S_{21} = \frac{S_{21}(f_0)}{1 + jQ_L\delta}$$

$$\angle S_{21} = -\arctan(Q_L\delta) \quad \tan(\angle S_{21}) = Q_L\delta \approx \frac{2\Delta\omega}{\omega_0}$$

$$\frac{\Delta\omega}{\omega_0} \approx -\frac{1}{2Q_L} \tan(\angle S_{21}) \approx -\frac{\angle S_{21}}{2Q_L}$$

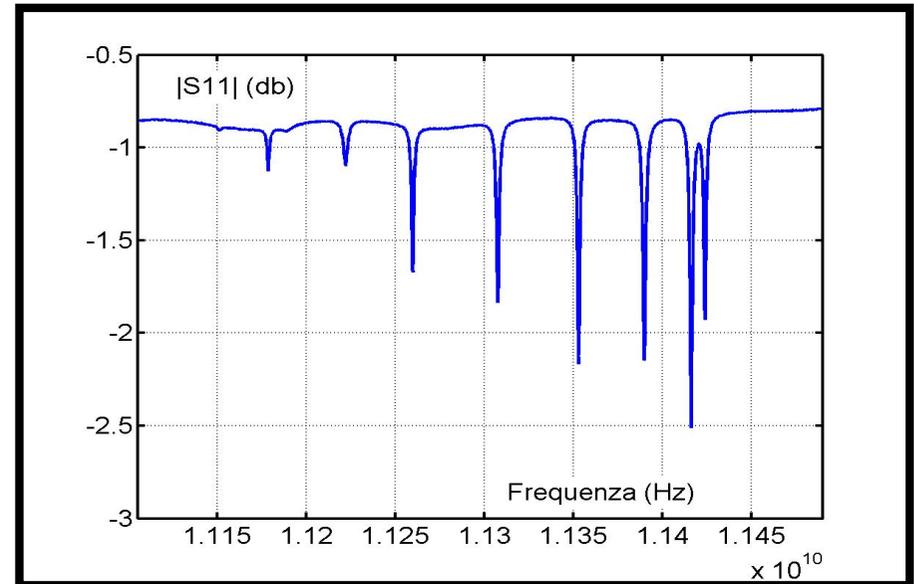
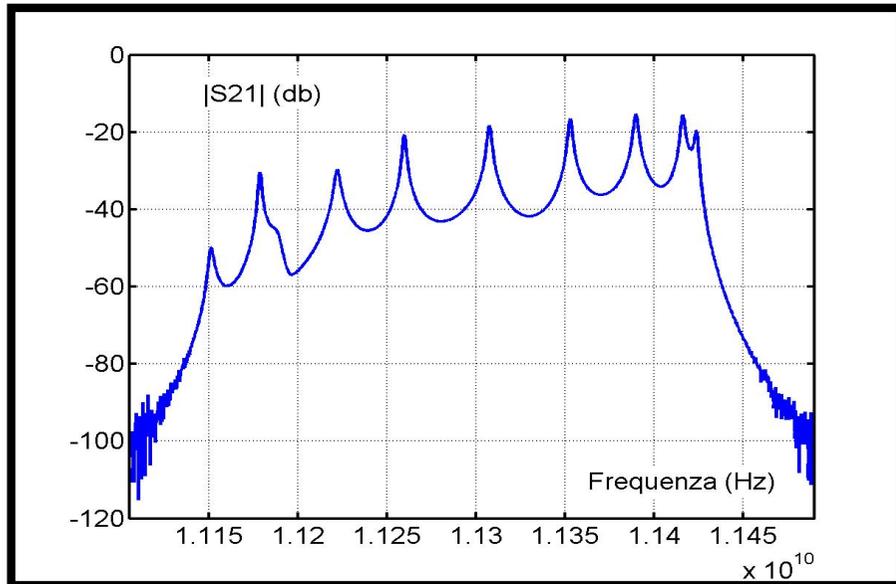
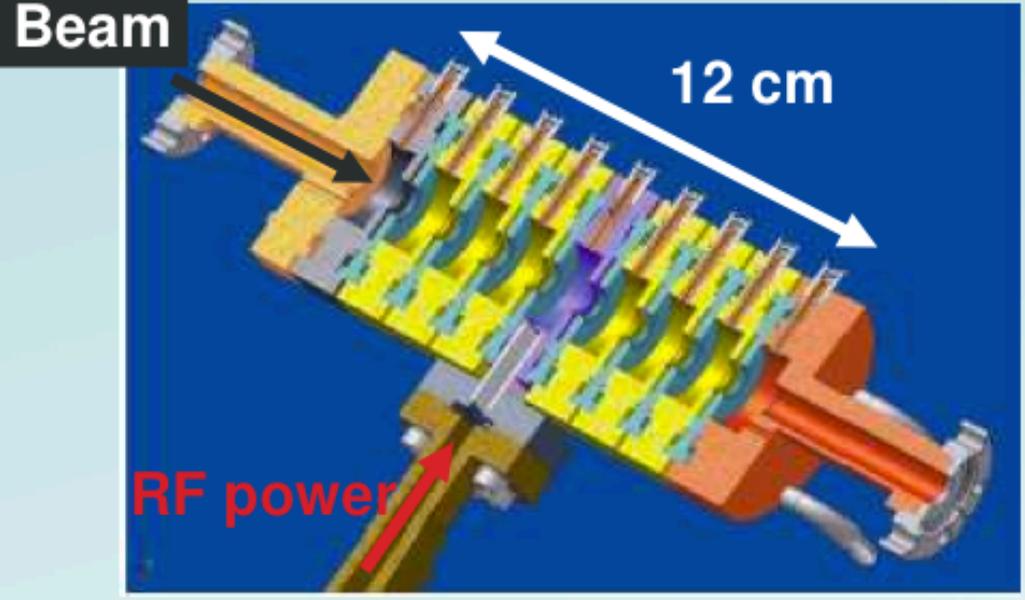
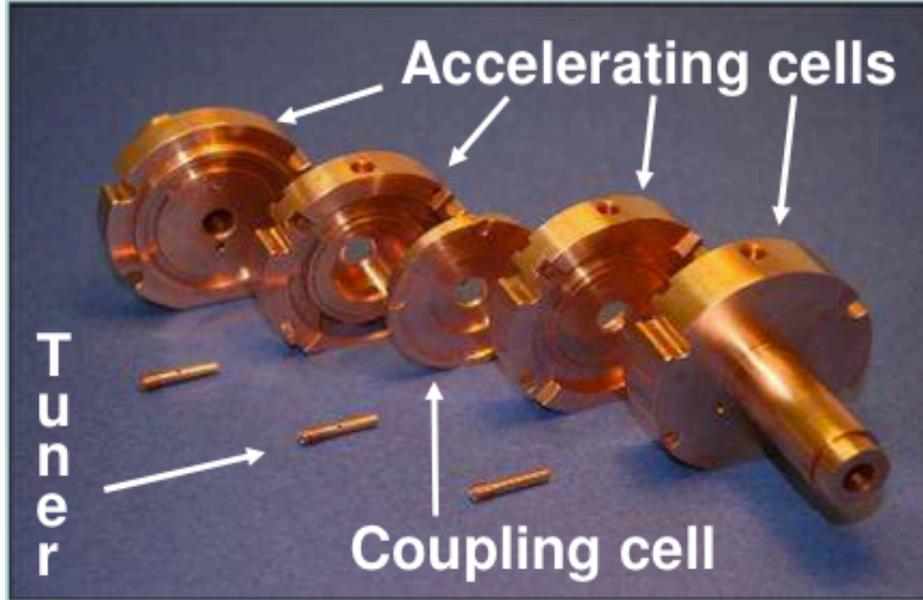
# Automatic field measurement (bead pull)



# Examples

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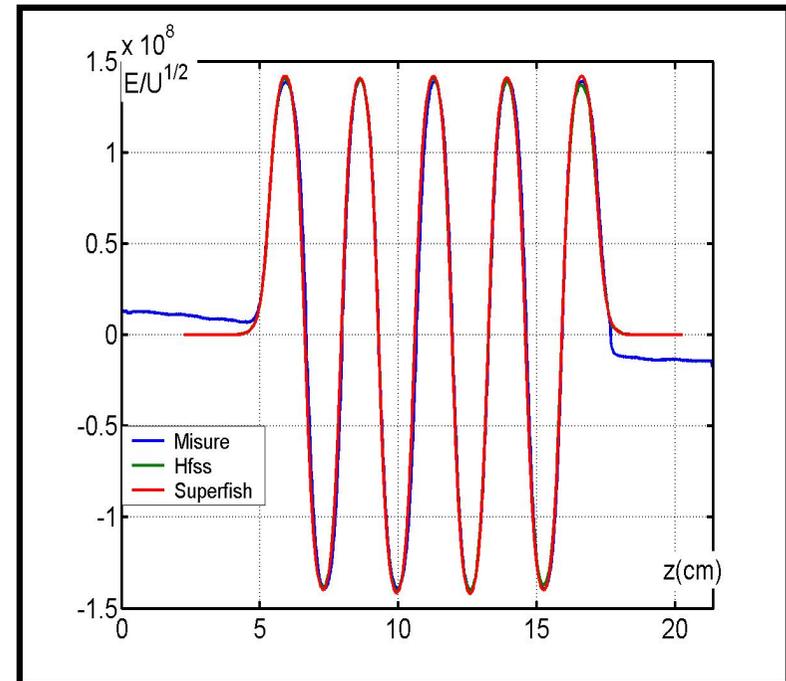
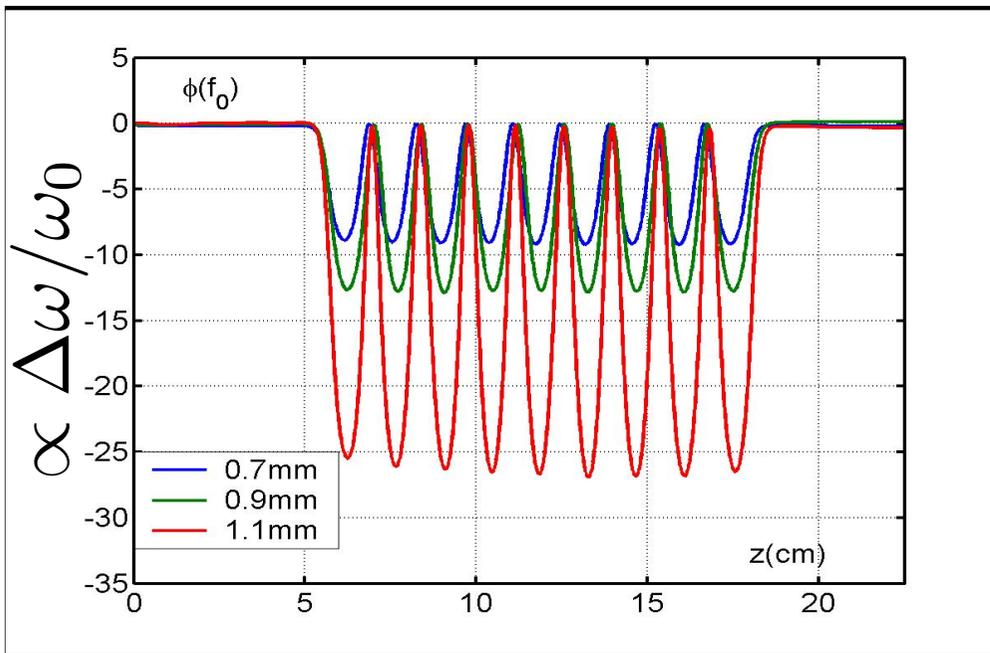
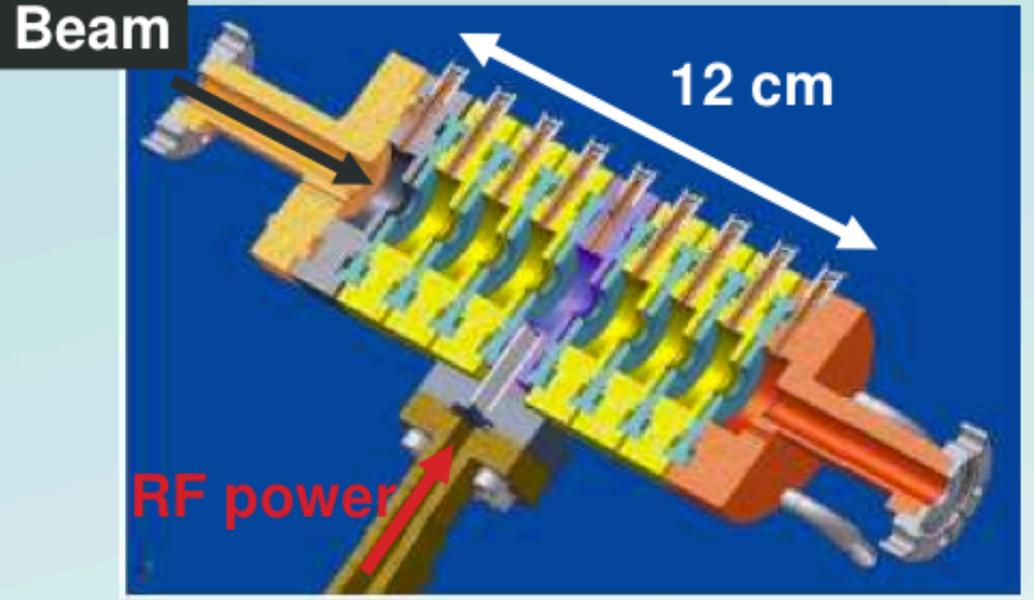
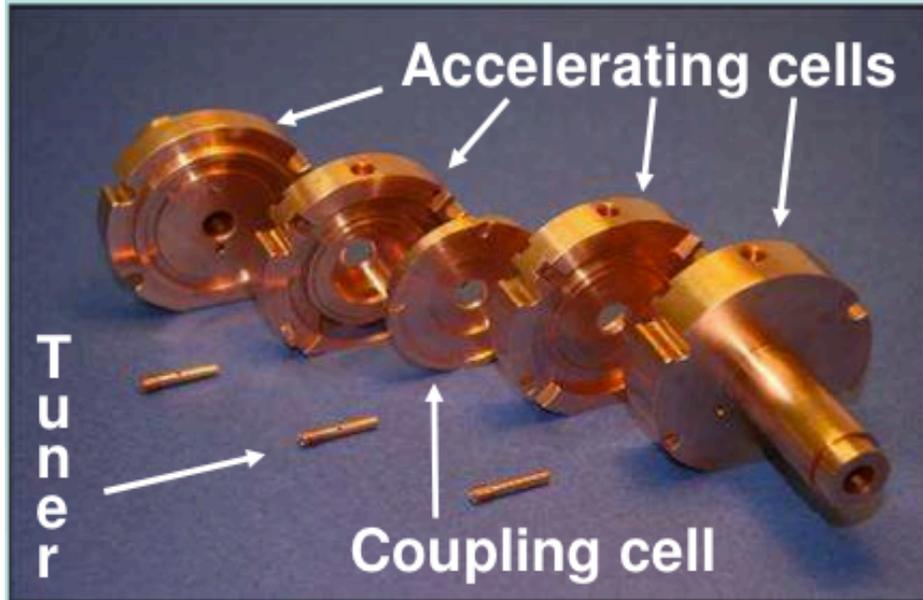
# Periodic cavity: S parameters



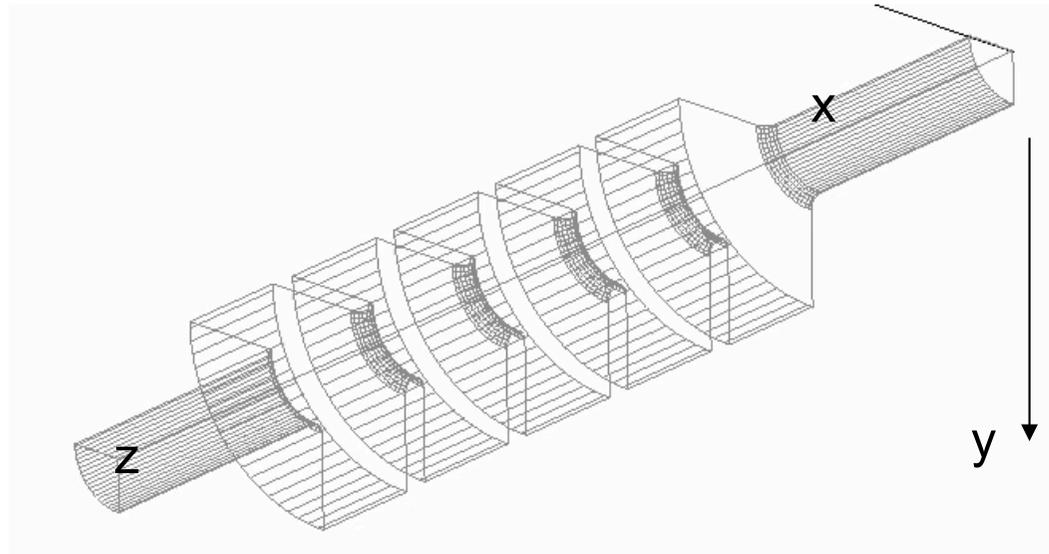
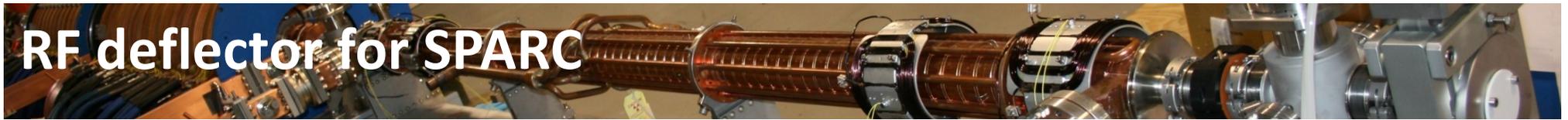
Courtesy of L. Ficcadenti

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# Periodic cavities: field on axis

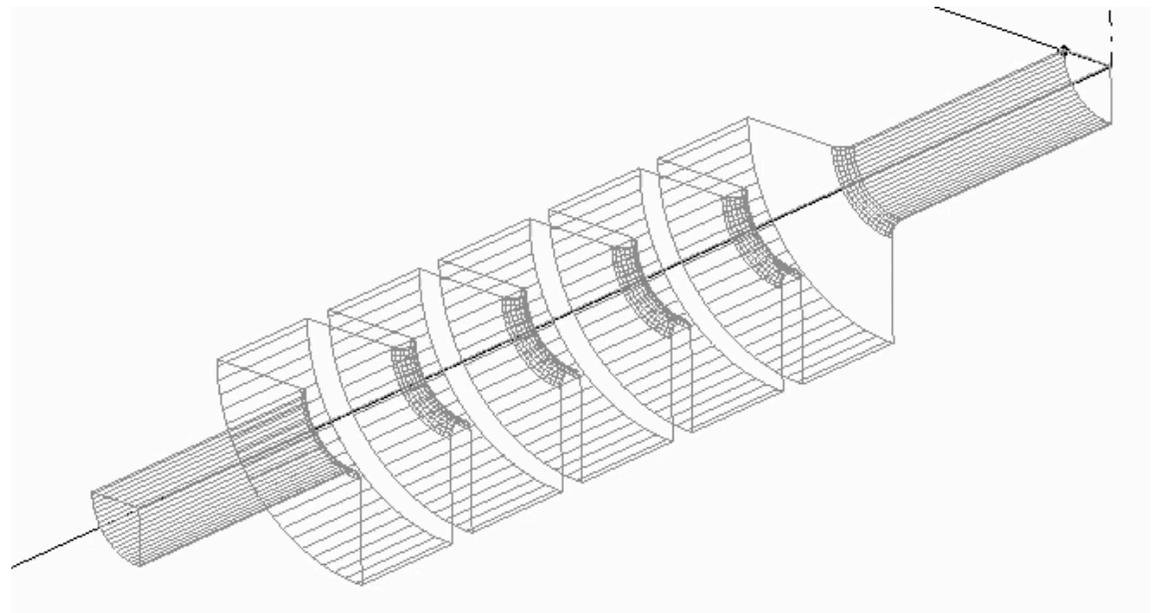


# RF deflector for SPARC

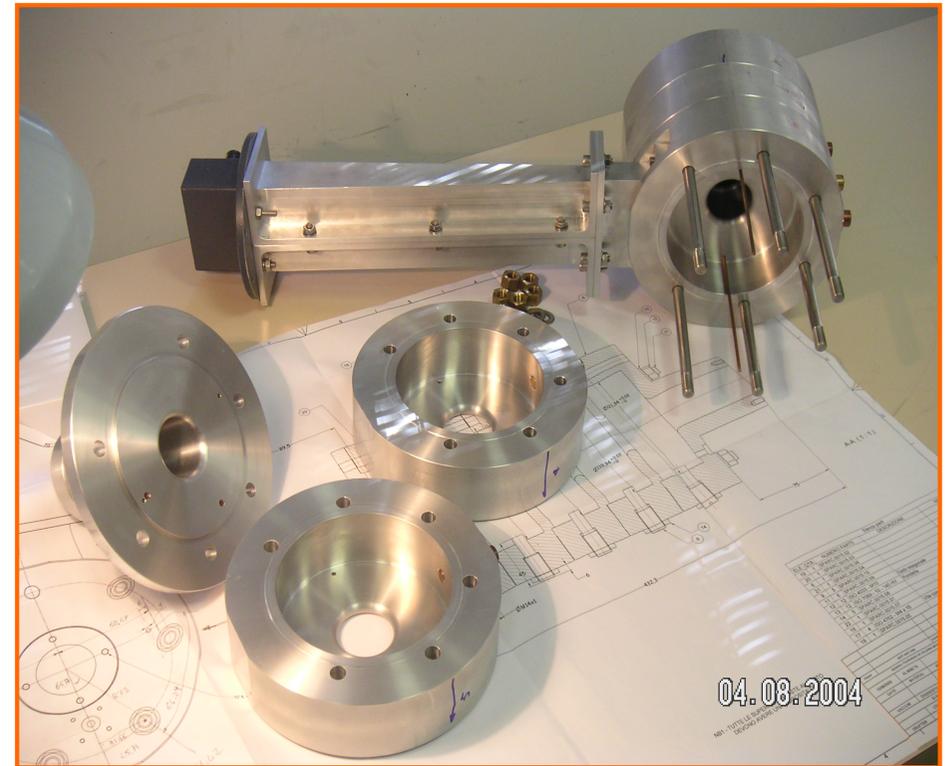
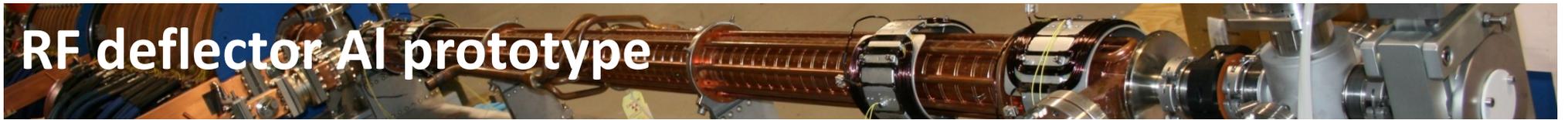


- Campo E sull'asse
- Modo  $\pi$  (TM<sub>110</sub>-like)

- Campo H sull'asse
- Modo  $\pi$  (TM<sub>110</sub>-like)



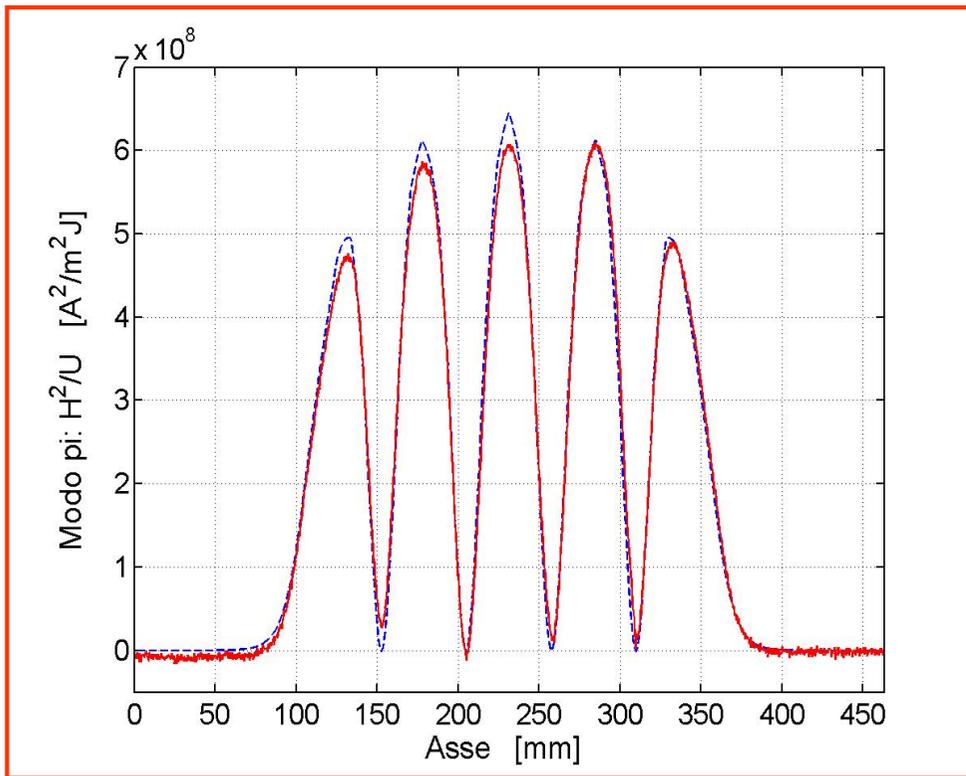
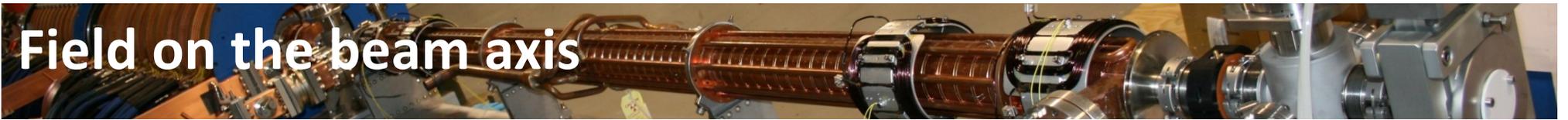
# RF deflector Al prototype



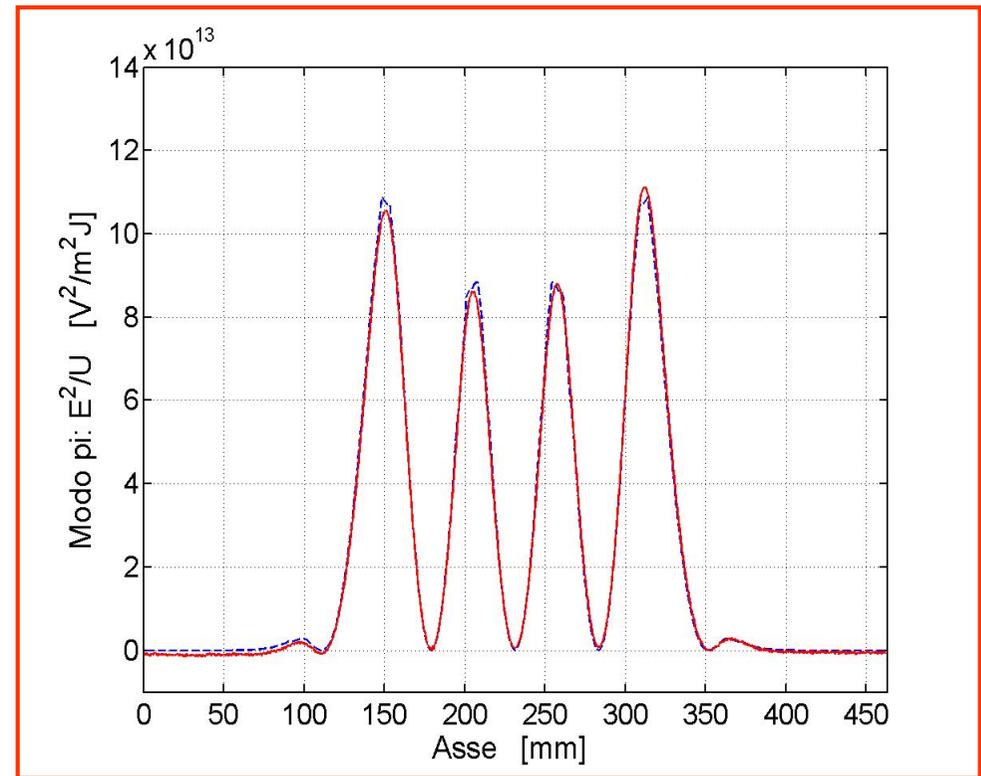
Courtesy of L. Ficcadenti

Andrea.Mostacci@uniroma1.it

# Field on the beam axis

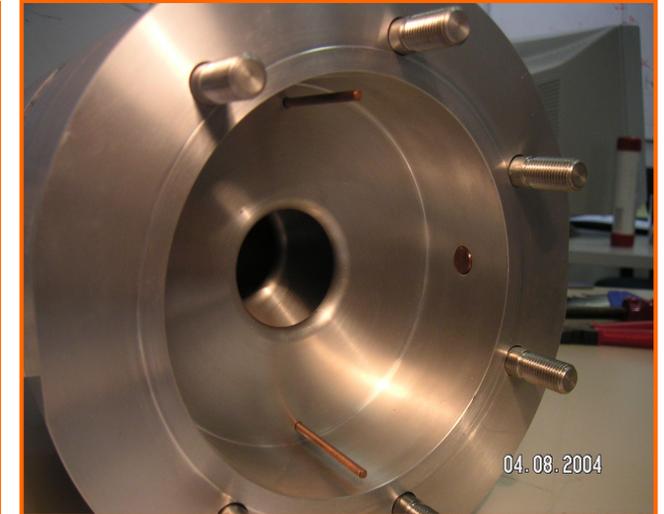
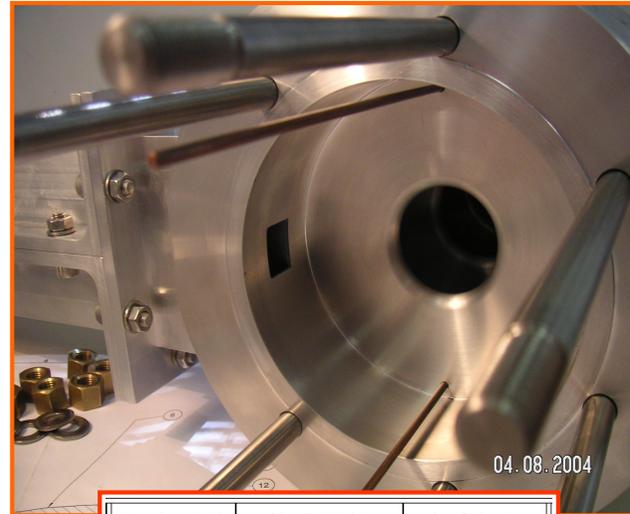


**Simulations (HFSS)**

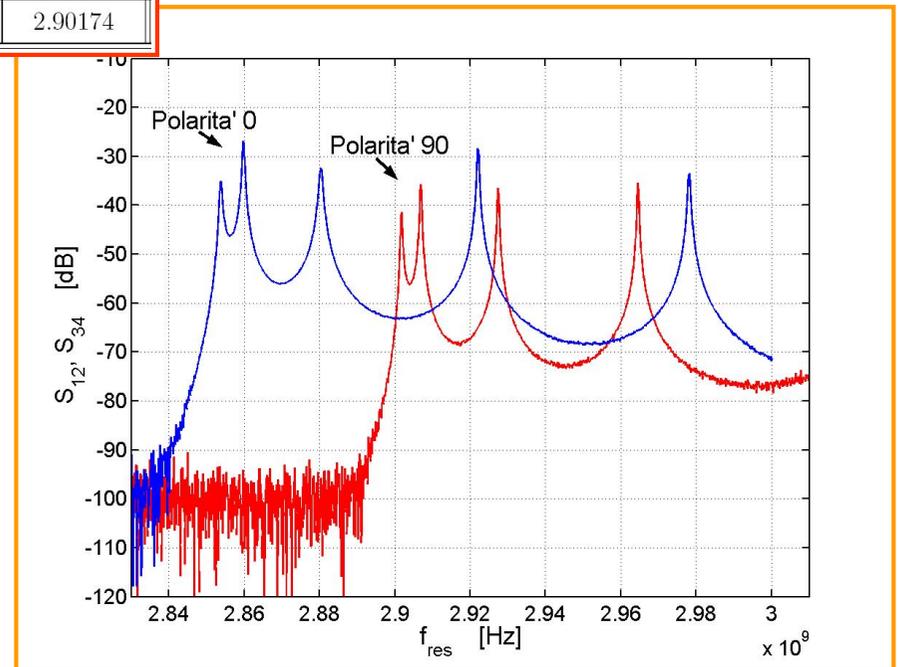
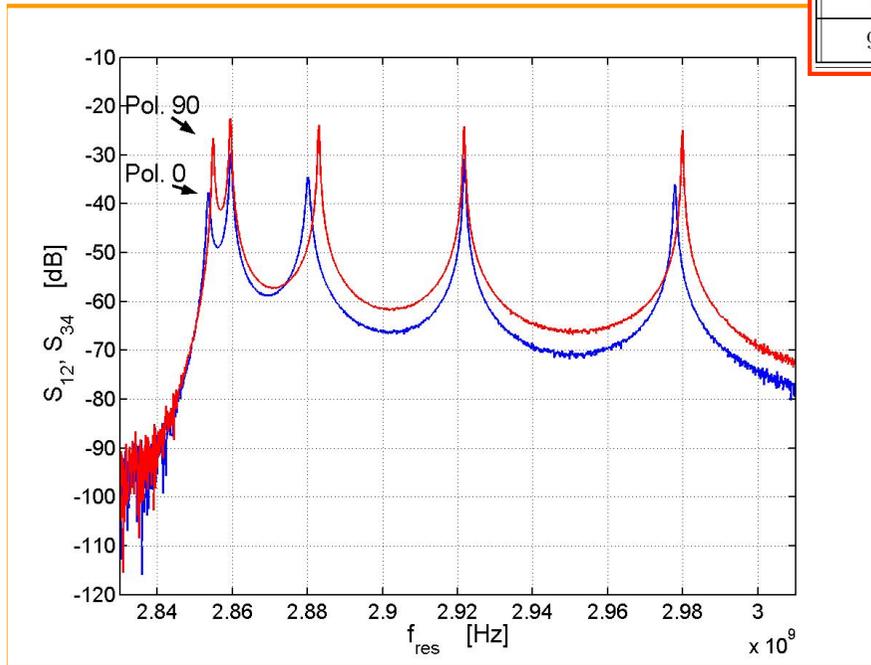


**Measurements (bead pull)**

# RF deflector field polarities



Polarità modo $\pi$	$f_{res}[GHz]$ senza barre	$f_{res}[GHz]$ con barre
0°	2.85358	2.85379
90°	2.85482	2.90174



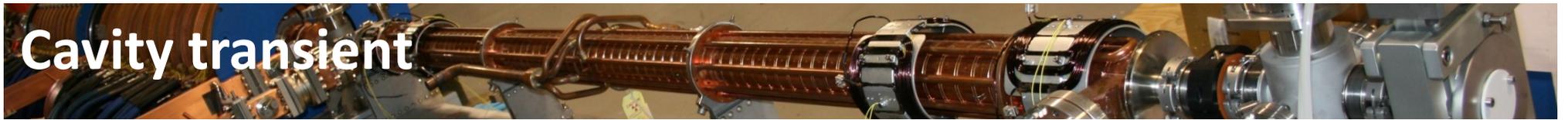
Courtesy of L. Ficcadenti

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# **Cavity transient**

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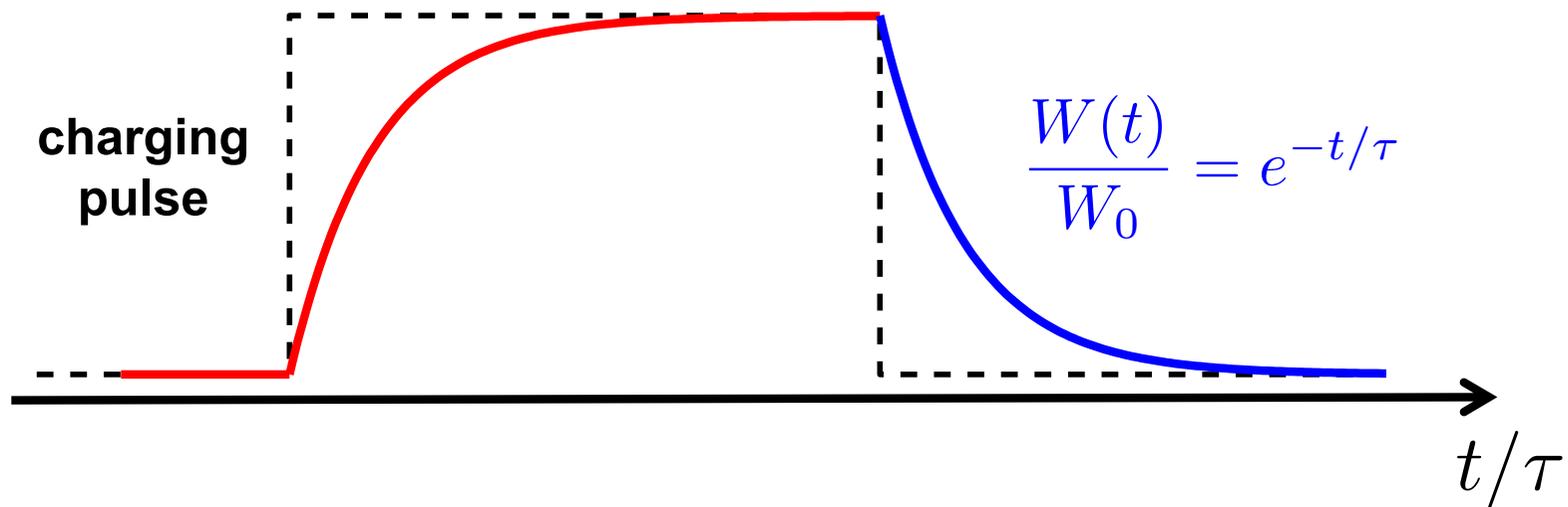
# Cavity transient



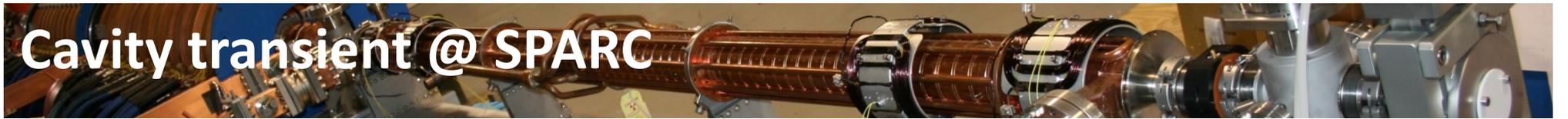
$$-dW = P_d dt = \frac{\omega_0 W}{Q_0} dt$$

$$\frac{W(t)}{W_0} = 1 - e^{-t/\tau}$$

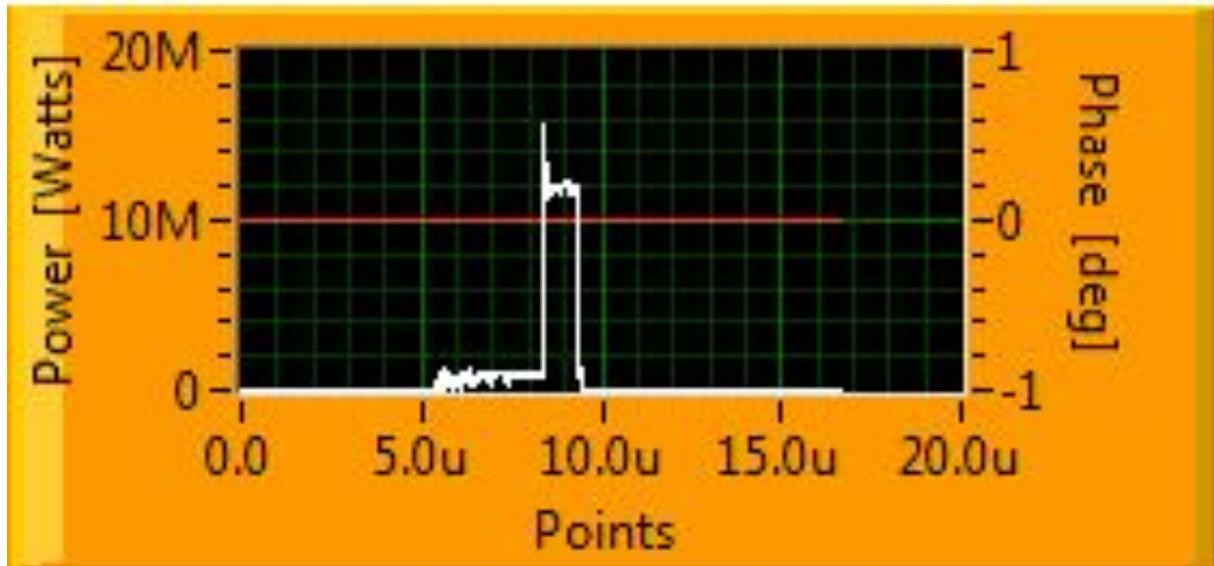
$$\tau = \frac{Q_0}{\omega_0} = \frac{Q_0}{2\pi f_0}$$



# Cavity transient @ SPARC



DGUN\_FRW



GunProbe

