# Periodic Signals

## Fourier series for periodic signals

$$\boxed{x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\Omega t + b_n \sin n\Omega t)}$$

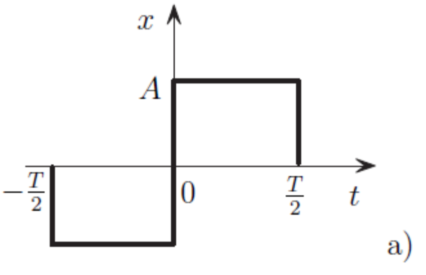
$$\boxed{a_n = \frac{2}{T}\int_T x(t) \cos n\Omega t \; dt \,, \quad b_n = \frac{2}{T}\int_T x(t) \sin n\Omega t \; dt}$$

Even Signals

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\Omega t$$

Odd Signals

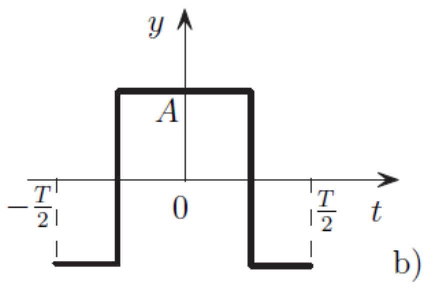
$$x(t) = \sum_{n=1}^{\infty} b_n \sin n\Omega t$$




Odd square wave

$$b_{2m} = 0\,, \qquad b_{2m+1} = \frac{4\,A}{\pi(2m+1)} \qquad (m = 0, 1, \dots\,)$$

$$\boxed{x(t) = \frac{4\,A}{\pi}\left(\sin \Omega t + \frac{1}{3}\sin 3\Omega t + \frac{1}{5}\sin 5\Omega t + \dots\right)}$$




Even square wave

$$a_0 = a_{2m} = 0\,, \qquad a_{2m+1} = (-1)^m \frac{4\,A}{\pi(2m+1)} \qquad (m = 0, 1, \dots)$$

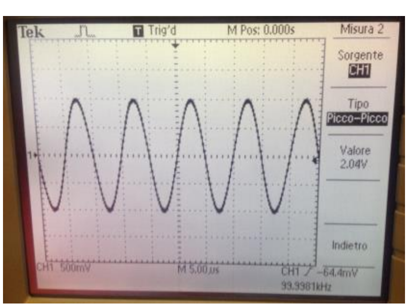
$$\boxed{y(t) = \frac{4\,A}{\pi}\left(\cos \Omega t - \frac{1}{3}\cos 3\Omega t + \frac{1}{5}\cos 5\Omega t + \dots\right)}$$

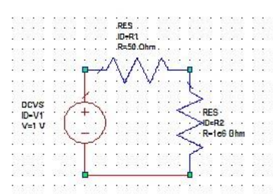
# LF Generator + Oscilloscope

It is set on the LF generator Vout = 1 V

$f_0 = 99.9981$ kHz

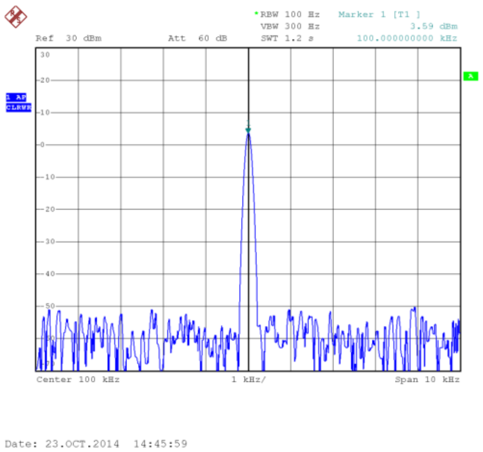
$V_{pp} = 2.04$ V

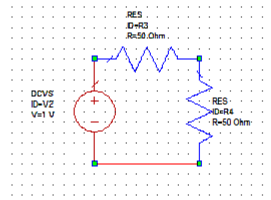



$$V_{R2} = \frac{R2}{R1 + R2}\,V = [R2 = 1M\,\Omega >> R1 = 50\,\Omega] \cong V\,;$$

# LF Generator + SPA




$$V_{R4} = \frac{R4}{R3 + R4} V = \frac{1}{2} V ;$$

# Link between OX and SPA

- The SPA measures the signal strength at 50 ohms. To evaluate this power when 1 V is set on the LF generator we must make the following considerations.
- The voltage read on the oscilloscope is the peak-to-peak voltage ($V_{pp}$). In order to calculate the power, the peak voltage ($V_p$=$V_{pp}$/2) and its effective value ($V_{eff}$=$V_p/\sqrt{2}$) must be calculated. As noted above it will be necessary to divide again by 2 due to partition losses. So the final power calculation will be:

- $V_p = \frac{V_{pp}}{2} = 1.020\ V$
- $V_{RMS} = \frac{V_p}{\sqrt{2}} = 0.721\ V$
- $V_x = \frac{V_{RMS}}{2} = 0.361\ V$
- $P = \frac{(V_x)^2}{50}$ = 0.00260 W = 2.6 mW = 4.14 dBm

# Amplitude and frequency modulation

## Amplitude Modulation

$$\boxed{V(t) = A_C\,[1 + a\cdot m(t)]\cos(2\pi f_C t + \vartheta)}$$

$A_c$: carrier amplitude
$f_c$: carrier frequency
a: modulation index ($0 \leq a \leq 1$)
m(t): normalised modulating signal
$A_c[1+m(t)]$: envelope amplitude

carrier

Lateral Bands

$$V(t) = A_C \cos(2\pi f_C t) + A_C\, a\, m(t) \cos(2\pi f_C t) = v_c(t) + v_s(t)$$

$$V(f) = V_c(f) + V_s(f)$$

# Amplitude Modulation

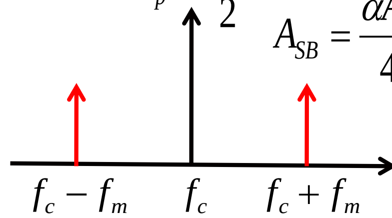
**Sinusoidal modulation**

$$v(t) = A_c \cos(2\pi f_c t) + A_c a \cos(2\pi f_m t)\cos(2\pi f_c t)$$

$$v(t) = A_c \cos(2\pi f_c t) + \frac{A_c a}{2}\left[\cos\left(2\pi\left(f_c + f_m\right)t\right) + \cos(2\pi\left(f_c - f_m\right)t)\right]$$

$$A_p = \frac{A_c}{2} \qquad A_{SB} = \frac{\alpha A_c}{4}$$

In addition to the carrier we have two side bands centered on the sum and the difference frequencies between that of the carrier and of the modulating signal. Both have an amplitude equal to that of the carrier for half the modulation inde x




$$\boxed{a = \frac{2A_{SB}}{A_p}}$$

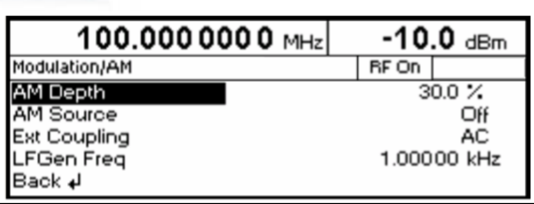
$$A_{SB}(dB) - A_p(dB) = 20\log(\frac{a}{2})$$

# Amplitude Modulation

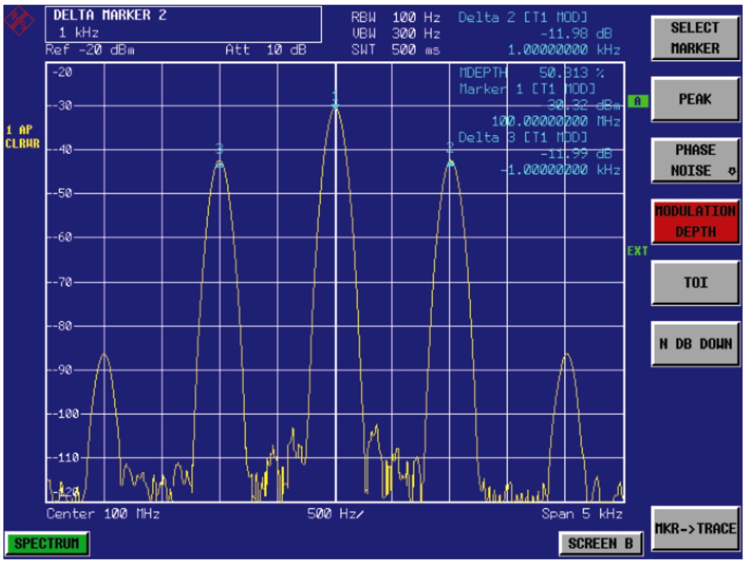
$$V(t) = A_C\,[1 + a\cdot m(t)]\cos(2\pi f_C t + \vartheta)$$

$A_c$: carrier amplitude
$f_c$: carrier frequency

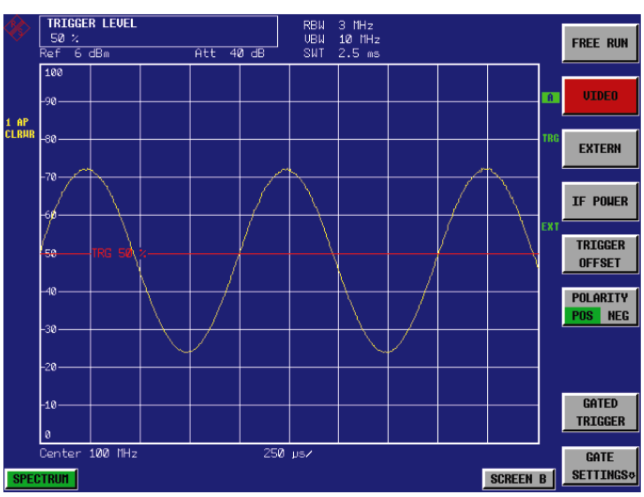
a: modulation index ($0 \le a \le 1$)
m(t): modulating signal



# Modulation Index measurement




# Time Domain Visualization




fc=100 MHz
fm=1 kHz

SETTING
Centr. Freq. 100 MHz
SPAN 0
RBW > BW
RANGE LINEAR
VIDEO TRIGGER

# Frequency Modulation

$$\boxed{v(t) = A\cos\left[2\pi f_c t + m\cos\left(2\pi f_m t\right) + \phi\right]}$$

A: carrier amplitude
$f_c$: carrier frequency

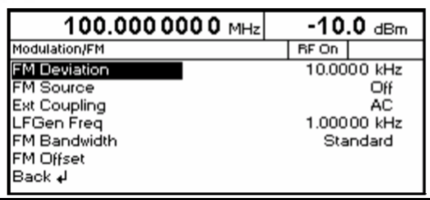
m: modulation index

$\Delta f = m f_m$ Max frequency deviation

$$\boxed{m = \frac{\Delta f}{f_m}}$$

Signal Bandwidth

$$\boxed{B = 2(m + 1)f_m}$$




# Frequency Modulation

$$v(t) = A\cos\left[2\pi f_c t + m\cos\left(2\pi f_m t\right) + \phi\right]$$

A: carrier amplitude
$f_c$: carrier frequency

m: modulation index

$$\boxed{v(t) = A\sum_{n=-\infty}^{+\infty} J_n(m)\cos\left[2\pi(f_c - n f_m)t + \phi\right]}$$

$$J_{-n}(m) = (-1)^n J_n(m)$$

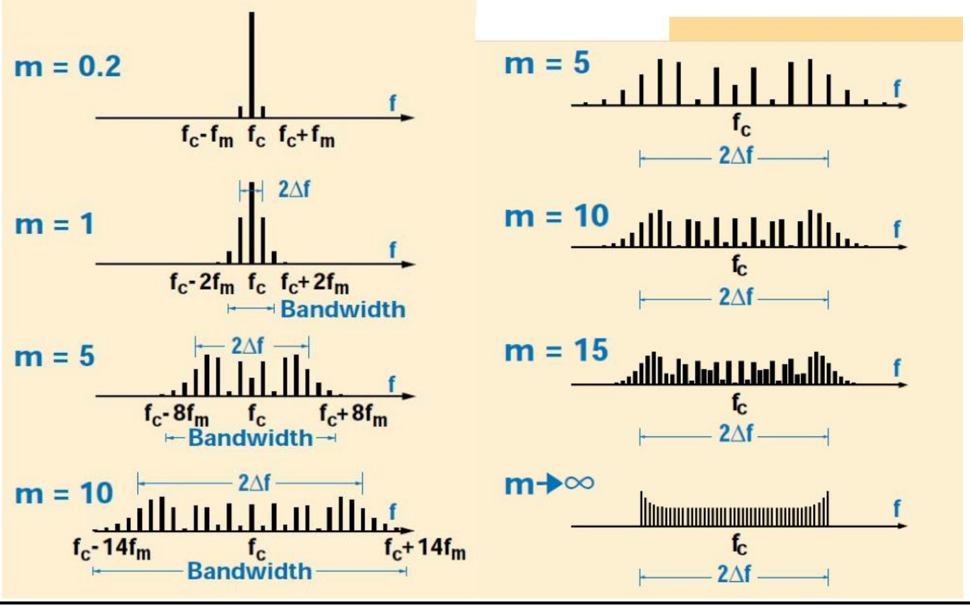
$$J_n(m) \approx 0 \quad if \quad n > m+1$$

$B = 2(m + 1)f_m$

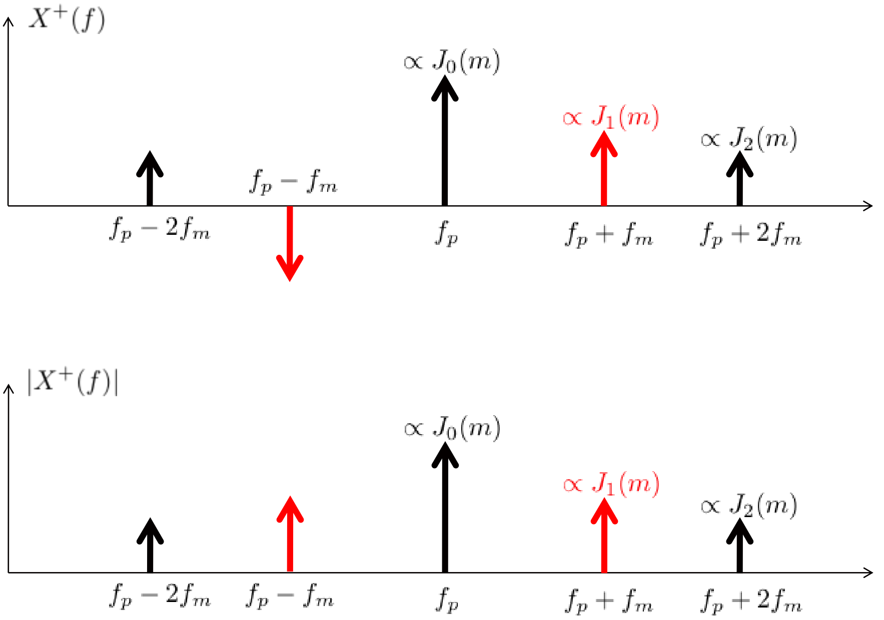
Carson formula

$$J_0(2.4) = 0 \quad \rightarrow$$

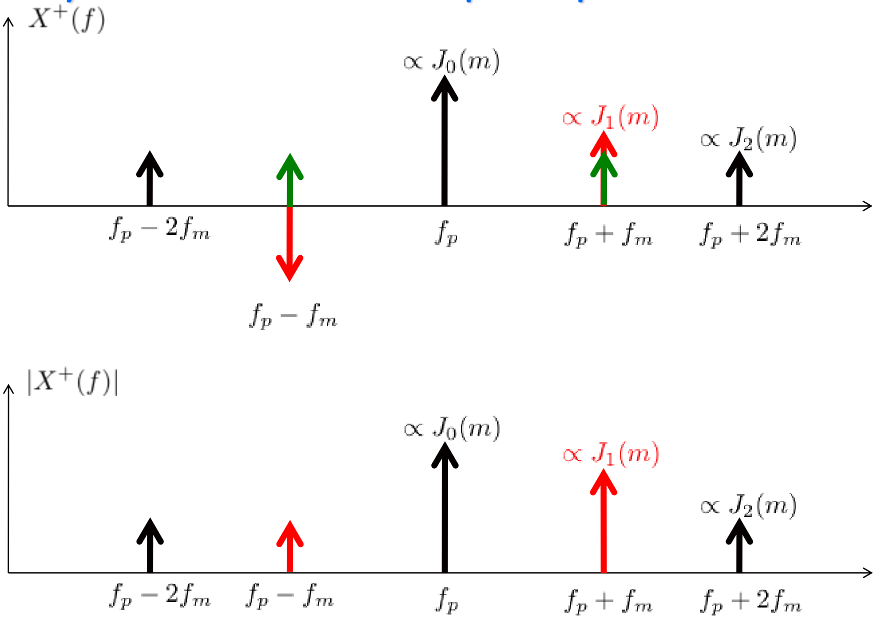
# Frequency Modulation




# Frequency modulation on the SPA



## Frequency Modulation with Superimposed AM Modulation

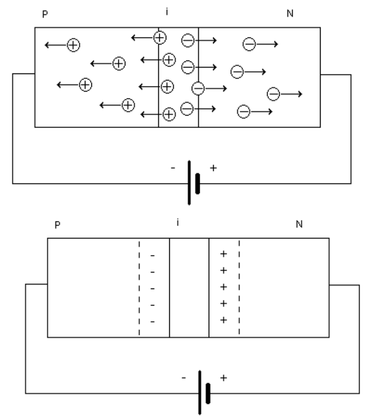


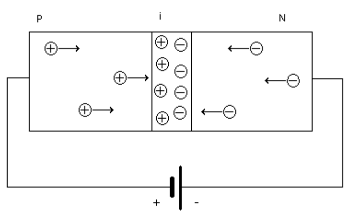

# Step recovery Diodes

# Step recovery Diodes

- Step recovery diodes are p-i-n diodes, where the intrinsic region is designed to have an average life time of the charges before recombination as long as possible.

Reverse Polarization

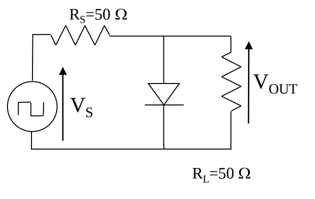


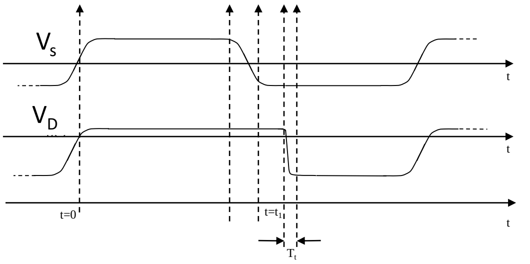




Direct Polarization

# Step Recovery Diodes

- When the diode is biased in direct electrons and holes enter the intrinsic region (accumulation) there is therefore a direct current in the diode.
- When the external voltage is reversed, the current is also reversed. The intrinsic region begins to empty (emptying) but, due to the presence of the accumulated charges, the voltage at the ends of the diode initially remains at the value that it had in direct.
- When all the charges present in the intrinsic region have been eliminated, the voltage at the ends of the diode is suddenly reversed and the diode is found in reverse polarization (transition and inversion).
- Since the switching speed is quite high, the diode is able to generate a rather steep voltage front, or a signal with a high bandwidth occupation.



# Transition Times

- The transition time, ie the time required to reverse the voltage across the diode, is related to the frequency band of the diode.
- Considering only the diode, limitations to the response speed are linked to the parasitic elements of the diode, that is to the junction capacity and to the resistance of the semiconductor.
- Therefore, by representing the diode by means of a capacitance and a resistance, the introduced time constant is given by $\tau$=RC

$$\boxed{f \le \frac{1}{2\pi\tau} = \frac{1}{2\pi RC}}$$ cut-off Frequency

The diode cut-off frequencies are of the order of 300 - 350 GHz.

# Device