

Cavity Parameters

There are a variety of parameters associated with resonant modes in a beamline rf cavity that have utility to design engineers, and a list of those directly computed by Analyst eigensolvers are shown in the following table.

Quantity	Expression	Units Reported	Output By	Notes
Resonant frequency (F _r)	---	MHz	All	
Mode energy (W)	$W = \epsilon_0 \int_V \vec{E} \circ \vec{\epsilon}_r \vec{E} dV$ <p>E is the electric field, and the integral is taken over the cavity volume. The relative permittivity can be a scalar or a diagonal tensor (defaults to unity).</p>	Joules	All	By default the mode fields are normalized so that this quantity is numerically equal to the free-space permittivity [1].
Voltage (V)	$V(r, \theta) = \int E_z(r, \theta, z) e^{j\omega_r z / \beta c} dz$ <p>Omega is resonant frequency, and beta is the particle beta (set by user). Integration is along particle path that is assumed to be axially directed (z), and a some fixed location in the transverse plane (r, theta).</p>	Volts	None	Not output. Used to compute other quantities. The axial (z) direction is determined by the integration path set by the user.
Effective surface resistance (R _s)	$R_s = \sqrt{\frac{\omega_r \mu_0}{2\sigma}}$ <p>Sigma is the bulk conductivity of the metal.</p>	Ohms	All	Used to compute wall power loss.
Wall power loss (P _l)	$P_l = \frac{R_s}{2} \int_E \vec{H} ^2 dS$ <p>H is the magnetic field, and the integral is taken over all external conducting surfaces.</p>	Watts	All	
Shunt impedance (R)	$R(r, \theta) = \frac{ V(r, \theta) ^2}{P_l}$	Ohms	All	
Quality Factor (Q)	$Q = \frac{\omega W}{P_l}$	---	All	
R/Q	$\frac{R(r, \theta)}{Q}$	Ohms	All	
Geometric factor G	<p style="text-align: center;">G = R_s Q</p> <p style="text-align: center;">Conductivity-independent</p> <p>R/Q*G gives a figure of merit for power efficiency in superconducting cavities</p>	Ohms	OM2P OM3P	

Power flow (P)				
Re[Transit – time factor] (T)	$T = \frac{\Re \left[\int_{z=0}^L E_z(z) e^{j\omega_r z / \beta c} dz \right]}{\left \int_{z=0}^L E_z(z) dz \right }$	---	OM2P (RZ)	The zero-phase point of the electric field is taken to be the midpoint of the integration path (unless an axial symmetry plane is used, in which case the zero phase point is on the symmetry plane).
Im[Transit – time factor] (S)	$S = \frac{\Im \left[\int_{z=0}^L E_z(z) e^{j\omega_r z / \beta c} dz \right]}{\left \int_{z=0}^L E_z(z) dz \right }$	---	OM2P (RZ)	(see note above)
Kick factor (K)	$K(r, \theta) = \frac{ V(r, \theta) ^2}{4 \frac{\omega_r}{\beta c} W r^2 d}$ <p>d is the integration path length (cell length), and beta is assumed to be unity (unless otherwise specified by the user).</p>	V/(pC*m ²)	All [2]	
E _{acc}	V(0,0)/L (See definition of V above)	V/m	OM2P (RZ)	Integral of Ez along axis divided by cavity length.
Peak E-field (E _p)	---	Volts/m	All	This is the peak field anywhere in the cavity.
Peak B-field (B _p)	---	Tesla	All	This is the peak field anywhere in the cavity.
Peak accelerating field (E _a)	---	Volts/m	OM2P (RZ)	Peak axial E field on axis. Location also output.
Peak axial magnetic field on axis (H _a)	---	Amps/m	OM2P (RZ)	Location also output.
Peak surface E-field (E _s)	---	Volts/m	OM2P (RZ)	Peak E field on conducting surface.

Peak surface H field (H_s)	---	Amps/m	OM3P, Omega3P	Peak H field on conducting surface.
E_s/E_a	$\frac{E_s}{E_a}$	---	Omega2	
E_s/E_{acc}	$\frac{E_s}{E_{acc}}$	---	OM2P (RZ)	

Notes:

[1] When symmetry planes are present and the solver can determine the fraction of the cavity that is actually being modeled, then the fields are normalized so that the energy in the entire cavity (not just the fraction modeled) is equal to the free space permittivity (8.854e-12).

[2] OM2P and Omega2 do not output this quantity for monopole modes.

The integration path that is used to determine the voltage is defined in the solver setup panel. For OM3P there are two paths, one for the voltage used in the shunt impedance calculation, and one for the voltage used in the kick factor calculation.