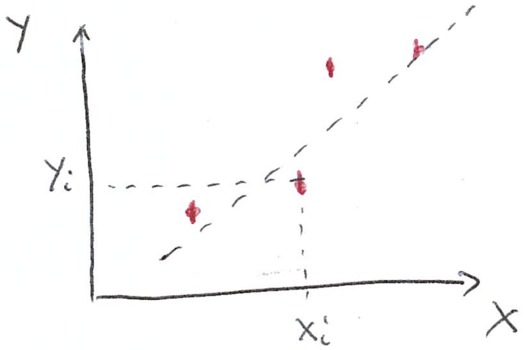
MINIMI QUADRATI E FIT LINEARI



QUALE È LA MIGLIOR RETTA ?

$$f(X_i, p, q) = p X_i + q$$

TRASCURO LE $u(Y_i)$

$$\chi^2 = \sum_{i=1}^{N} \left[Y_i - f(X_i, p, q) \right]^2$$

→ QUALI p E q MINIMIZZANO χ^2 ?

$$\left.\begin{aligned} \frac{\partial \chi}{\partial p} &= 0 \\ \frac{\partial \chi}{\partial q} &= 0 \end{aligned}\right\} \quad \begin{aligned} p &= \dots \\ q &= \dots \end{aligned}$$

$u(p) = ?$

$u(q) = ?$

PER $u(p)$ e $u(q)$ CONSIDERO

→ INCERTEZZE DELLE $u(Y_i)$ SE DISPONIBILI/AFFIDABILI

→ $$\sigma = \sqrt{\frac{\sum_{i=1}^{N} \left[Y_i - (p X_i + q) \right]^2}{\underbrace{N-2}_{\text{\# GRADI DI LIBERTÀ}}}}$$

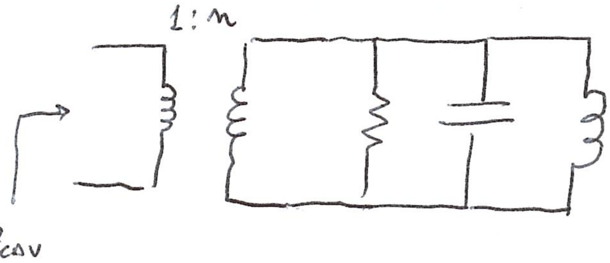
RICORDARE :

LINEARIZZAZIONI E SCALE LOGARITMICHE RENDONO ~~[illegible]~~ ~~[illegible]~~ FIT LINEARE MOLTO UTILE

MISURE IN RIFLESSIONE DI CAVITA'

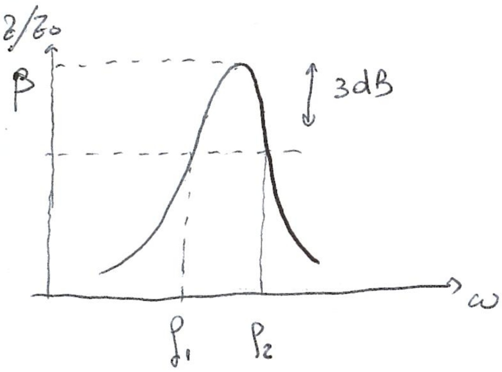
→ CAVITA' ACCESSIBILI DA UNA SOLA PORTA

RICAPITOLO



$\beta = \frac{1}{n^2}$

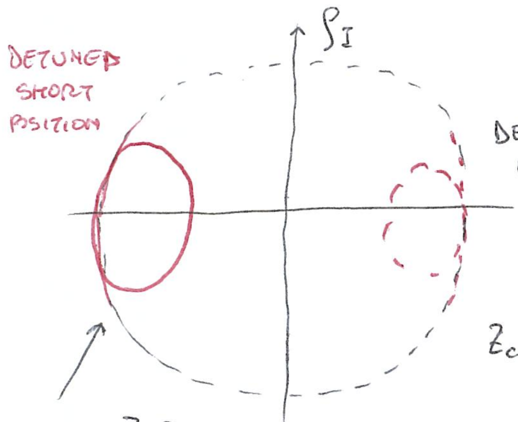
$$Z_{CAV} = Z_0 \frac{\beta}{1 + jQ_0\delta} \qquad \delta = \frac{f}{f_0} - \frac{f_0}{f} \simeq \frac{2(f-f_0)}{f_0}$$



$$Q_0 = \frac{f_0}{f_2 - f_1} \qquad |Z|(f_i) = \frac{max}{\sqrt{2}}$$

⇒ AUTOMATIC Q MEASUREMENT
SU $Z(\omega)$

QUESTO VALE SE IL MODELLO È Z_{CAV} ⇒ DETUNED SHORT POSITION



$$Z_{CAV} = Z_0 \left(\frac{1}{\beta} + j \frac{Q_0}{\beta} \delta \right)$$

$$Z_{CAV} = \frac{Z_0 \beta}{1 + jQ_0\delta}$$

$\boxed{\text{I MODO}}$

CON IL PHASE OFFSET PORTARE NEL DETUNED SHORT POSITION E FARE LA CONVERSIONE IN Z_{CAV} REFL

$$\frac{Z(f_0)}{Z_0} = \beta \qquad Q_0 = \frac{f_0}{\Delta f_{3dB}}$$

$$\frac{Z_{CAV}}{Z_0} = \frac{\beta}{1 + j Q_0 \delta} = \frac{\beta}{1 + (Q_0 \delta)^2} \left[1 - j Q_0 \delta \right]$$

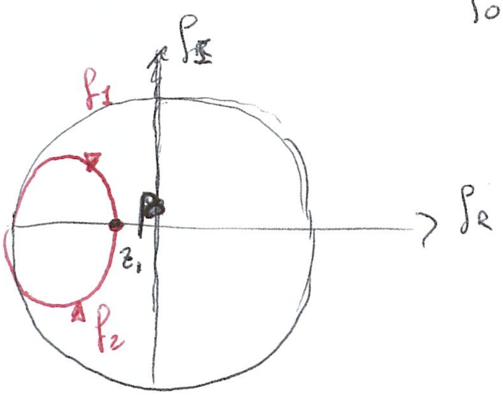
SE $\quad Re(z) = \pm Im(z) \Rightarrow Q_0 \delta = \pm 1 \qquad \delta = \dfrac{2(f - f_0)}{f_0}$

$$f_1 : \frac{2(f_1 - f_0)}{f_0} = \frac{1}{Q_0} \qquad f_2 : \frac{2(f_2 - f_0)}{f_0} = -\frac{1}{Q_0}$$

$$\frac{2(f_1 - f_0)}{f_0} - \frac{2(f_2 - f_0)}{f_0} = \frac{2}{Q_0}$$

$$\Downarrow$$

$$\frac{f_1 - f_2}{f_0} = \frac{1}{Q_0} \qquad Q_0 = \frac{f_0}{f_1 - f_2}$$

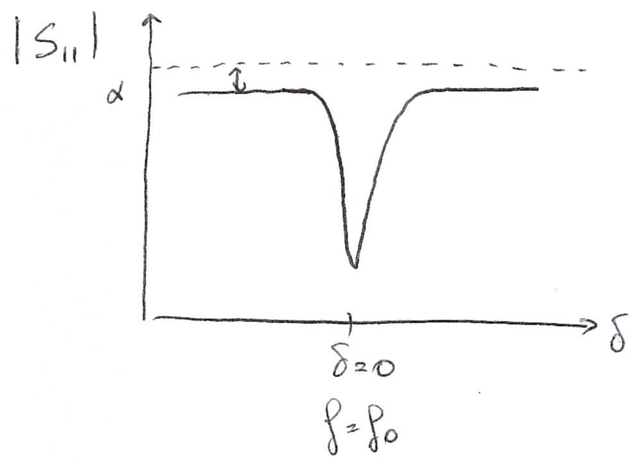


$$\beta = \frac{Z_1}{50}$$

$\boxed{\text{II MODO}}$ LETTURA CURSORI CARTA DI SMITH

IN RIFLESSIONE

$$S_{11} = \frac{Z_{CAV} - Z_0}{Z_{CAV} + Z_0} = \underset{\uparrow \text{ LOSS}}{\alpha} \, \frac{\beta - 1 - j Q_0 \delta}{\beta + 1 + j Q_0 \delta}$$



$$S_{11}(\rho = \rho_0)\Big|_{\alpha \simeq 1} = \frac{\beta - 1}{\beta + 1}$$

$$SWR = \frac{1 + |S_{11}|}{1 - |S_{11}|}$$

SE $\beta < 1$ $\quad SWR(\delta = 0) = \dfrac{1}{\beta}$

SE $\beta > 1$ $\quad SWR(\delta = 0) = \beta$

$\Big\}$ SE $\alpha = 1$

SE $\alpha \neq 1$ O NON SI PUO' ANDARE NELLA DETUNED SHORT POSITION

$$\chi^2 = \sum_{i=1}^{N} \left[S_{11}^{MEAS} - g(\delta, \alpha, \beta, Q_0) \right]^2$$

CON $g = \alpha \dfrac{\beta - 1 - j Q_0 \delta}{\beta + 1 + j Q_0 \delta}$ $\Rightarrow$ MATLAB FIT TOOL

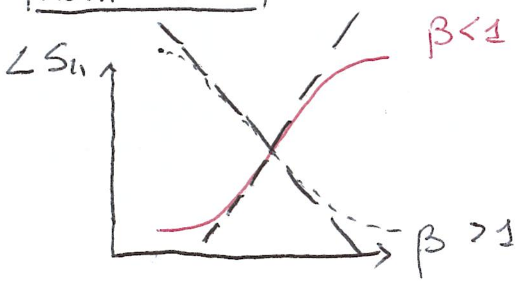
$\alpha \pm u(\alpha)$

$\beta \pm u(\beta)$

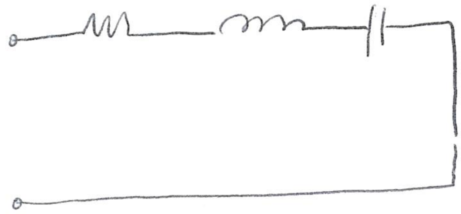
$Q_0 \pm u(Q_0)$

$\boxed{\text{III MODO}}$

$\boxed{\text{NOTA BENE}}$

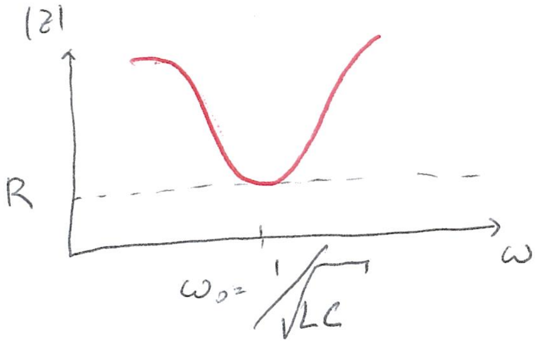


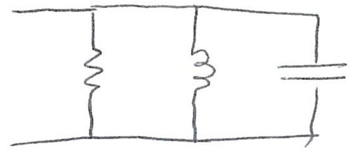
$$\angle S_{11} = \underbrace{\frac{4\beta}{1 - \beta^2} \frac{Q_0}{\rho_0}}_{\text{PENDENZA RETTA}} (\rho - \rho_0)$$



$$Z = R + j\omega L + \frac{1}{j\omega C}$$

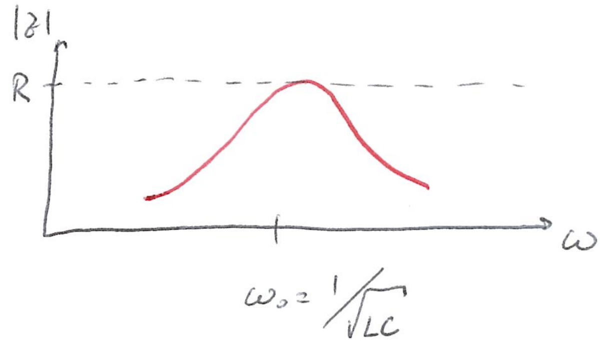
$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$





$$Z = \frac{R}{1 + j\frac{RC}{L}\left(\omega L - \frac{1}{\omega C}\right)}$$

$$|Z| = \frac{R}{1 + \left(\frac{RC}{L}\right)^2\left(\omega L - \frac{1}{\omega C}\right)^2}$$



$Im(Z) = 1$

QE: intersect with Z=j*50
Ohm in detuned open
position; equivalent to Y=j1
=> f3 and f4

QL: search for max / min
(Im(S11))
=> f1 and f2

$Q_L \delta = 1$

Q0: Re(Z) = Im(Z)
=> f5 and f6

si può fare con la conversione
Z from S11

f_3

R: read Re(S11)
at Min(abs(S11))

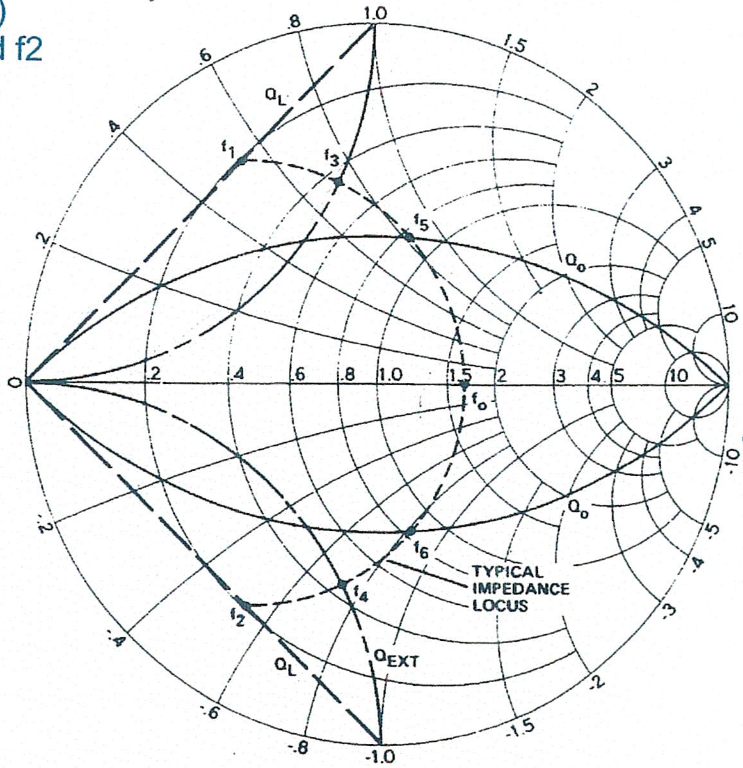


Fig. 30 Loci of Q_0, Q_L and Q_{EXT} of a resonator

$$Q_{EXT} = \frac{Q_0}{\beta}$$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

$$Q = \frac{f_0}{\Delta f} \quad (3\,\text{dB}). \tag{42}$$

Referring to Fig. 30 we obtain

$$\begin{aligned} Q_0 &= f_0/(f_5 - f_6) \\ Q_L &= f_0/(f_1 - f_2) \\ Q_{ext} &= f_0/(f_3 - f_4) \end{aligned} \tag{43}$$

Figure 30 shows the resonator in the "detuned short position", i.e. the impedance of the resonator far from f_0 approaches zero and f_1, f_2 are "half-power" points (intersection with straight lines connecting $\rho = 0$ and $\rho = \pm j$ respectively. For the external Q (f_3, f_4) we must turn the locus of the resonator into "detuned open position" ($\lambda/4$ transformer). This produces an intersection with the constant reactance circle $Z = Y/Z_0 = \pm j$. Finally the points f_5, f_6 (Q_0; $R_{cavity} = \pm X_{cavity}$) are read from the intersection of a cavity with two circles centered at $\pm j$ and having a radius of $\sqrt{2}$.

There are three ranges of the coupling factor β defined by

CALCOLO DI β

$$\frac{Z}{Z_0} = \frac{\beta}{1 + jQ_0\delta} \qquad S_{11} = \frac{\beta - 1 - jQ_0\delta}{\beta + 1 + jQ_0\delta}$$

$$SWR = \frac{1 + |S_{11}|}{1 - |S_{11}|}$$

@ RESONANCE $(\delta = 0)$

$$SWR = \frac{|\beta+1| + |\beta-1|}{|\beta+1| - |\beta-1|}$$

I MODO

VISUALIZZO LA CARTA DI SMITH E METTO IL CURSORE SULLA RISONANZA ($\delta = 0$, IMPEDENZA REALE)

LEGGO LA RESISTENZA IN Ω DAL CURSORE DELLA C. DI SMITH

$Z(\Omega)$

$$\beta = \frac{Z(\Omega)}{50\,\Omega}$$

II MODO

METTO IL CURSORE ALLA RISONANZA $(\delta = 0)$ E MISURO LA PARTE REALE DI S_{11}

$$R_e(S_{11}) = \frac{\beta - 1}{\beta + 1}$$

III MODO

MISURO LO SWR ALLA RISONANZA $\delta = 0$

Se $\beta < 1$ $\qquad SWR(\delta = 0) = \frac{1}{\beta}$

SE $\beta > 1$ $\qquad SWR(\delta = 0) = \beta$

MISURA DI Q_0

DETUNED SHORT POSITION

$$Z = \frac{\beta}{1+(Q_0\delta)^2} - j\,\frac{\beta Q_0 \delta}{1+(Q_0\delta)^2}$$

δ_1: $\quad Re(Z) = Im(Z) \qquad \beta = -\beta Q_0 \delta_1 \qquad \delta_1 = -\dfrac{1}{Q_0}$

δ_2: $\quad Re(Z) = -Im(Z) \qquad \beta = +\beta Q_0 \delta_2 \qquad \delta_2 = +\dfrac{1}{Q_0}$

$$\delta = \left(\frac{f}{f_0} - \frac{f_0}{f}\right) = x - \frac{1}{x} \simeq 2(x-1)$$

$$\delta_2 - \delta_2 \cong 2(x_2 - x_1) \cong \frac{2}{Q_0}$$

$$Q_0 = \frac{1}{x_2 - x_1} = \frac{f_0}{f_2 - f_1}$$

$x_1 = \dfrac{f_1}{f_0}$ TALE CHE $Re\, Z = Im\, Z$

$x_2 \quad \dfrac{f_2}{f_0}$ TALE CHE $Re\, Z = -Im\, Z$

- VALE SOLO PER Q_0 ALTI
- FARE LA VERIFICA PER VEDERE FINO A CHE Q_0 VALE CON MATHEMATICA

①

$$\rho = \alpha \frac{\beta - 1 - jQ_0\delta}{\beta + 1 + jQ_0\delta}$$

$$\angle\rho = \angle\alpha + \angle\left(\beta - 1 - jQ_0\delta\right) - \angle\left(\beta + 1 + jQ_0\delta\right)$$

$$= \angle\alpha + \varphi_1 - \varphi_2 \qquad\qquad \varphi = tg^{-1}\frac{y}{x}$$

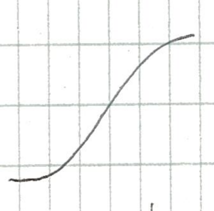
$$\varphi_1 = tg^{-1}\frac{Q_0\delta}{1-\beta} \qquad \varphi_2 = tg^{-1}\frac{Q_0\delta}{\beta+1}$$

SE $Q_0\delta \ll 1$ $\qquad tg^{-1}x \simeq x$

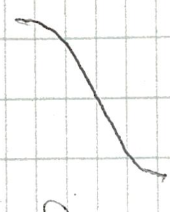
$$\varphi_1 - \varphi_2 = Q_0\delta\left(\frac{1}{1-\beta} - \frac{1}{\beta+1}\right) = Q_0\delta\,\frac{\beta+1-1+\beta}{1-\beta^2} =$$

$$= Q_0\delta\,\frac{2\beta}{1-\beta^2}$$

$\beta < 1$



$\beta > 1$



$$\delta = \left(\frac{f}{f_0} - \frac{f_0}{f}\right) = \frac{f^2 - f_0^2}{f_0 f} = \frac{(f-f_0)\,2f_0}{f_0 f} \approx \frac{2(f-f_0)}{f_0}$$

$$\angle\rho = \angle\alpha + \varphi_1 - \varphi_2 = \angle\alpha + Q_0\delta\,\frac{2\beta}{1-\beta^2} \simeq \angle\alpha + \frac{2\beta}{1-\beta^2}\,\frac{Q_0}{f_0}\,2(f-f_0)$$

PENDENZA $= \dfrac{4\beta}{1-\beta^2}\,\dfrac{Q_0}{f_0}$ $\rightrightarrows$ FIT LINEARE

DETUNED OPEN POSITION $\frac{Z(\omega)}{Z_0} = \frac{1}{\beta} + j \frac{Q_0}{\beta} \delta$

$$\rho = \frac{Z/Z_0 - 1}{Z/Z_0 + 1}$$

$$\rho = \left(\frac{1}{\beta} + j \frac{Q_0}{\beta} \delta - 1\right) \Big/ \left(\frac{1}{\beta} + j \frac{Q_0}{\beta} \delta + 1\right)$$

$$= \frac{1 - \beta + j Q_0 \delta}{1 + \beta + j Q_0 \delta}$$ DETUNED OPEN POSITION

$$\rho = - \frac{1 - \beta + j Q_0 \delta}{1 + \beta + j Q_0 \delta}$$ DETUNED SHORT POSITION

IL FIT SUL MODULO VA BENE QUALSIASI SIA LA POSIZIONE SULLA CARTA DI SMITH

IN GENERALE $S_{11} = e^{-j\omega\tau} S_{11}^{\text{DETUNED SHORT}}$

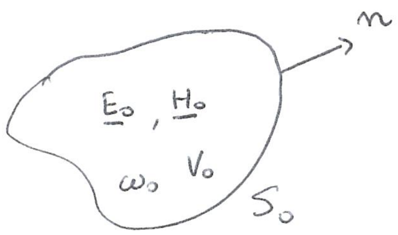
PERTURBAZIONE DELLA FORMA DI CAVITÀ

CASO PRATICO:

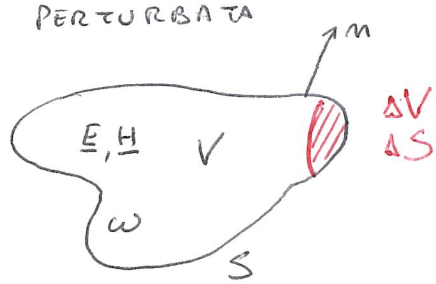
- INSERIMENTO DI PICCOLI OGGETTI METALLICI / DIELETTRICI IN UNA CAVITÀ
- MODIFICHE "LOCALI" DELLA FORMA DELLA CAVITÀ

VARIA LA FREQUENZA DI RISONANZA !!

CAVITÀ IMPERTURBATA



CAVITÀ PERTURBATA



(VEDI HENKE)

$$\nabla \times \underline{E}_0 = -j\omega_0 \mu_0 \underline{H}_0$$

$$\nabla \times \underline{H}_0 = j\omega_0 \varepsilon \underline{E}_0$$

$$\nabla \times \underline{E} = -j\omega \mu_0 H$$

$$\nabla \times \underline{H} = j\omega \varepsilon E$$

SCRIVENDO $\underline{H} \cdot (\nabla \times \underline{E}_0^*) - \underline{E}_0 \cdot (\nabla \times \underline{H}) = ----$ $+ \int_V$

$---$

$\Downarrow$

$$\omega - \omega_0 = -j \frac{\oint_{\Delta S} (\underline{E}_0 \times \underline{H}) \cdot d\underline{S}}{\int_V (\varepsilon \underline{E} \cdot \underline{E}_0 + \mu \underline{H} \cdot \underline{H}_0^*)\, dV}$$) POYNTING

$$\frac{\omega - \omega_0}{\omega_0} \approx \frac{\int_{\Delta V} (\mu |H_0|^2 - \varepsilon |E_0|^2)\, dV}{\int_{V_0} (\mu |H_0|^2 + \varepsilon |E_0|^2)\, dV} = \frac{\Delta W_m - \Delta W_e}{W_m - W_e}$$

ΔW_m VARIAZIONE DI ENERGIA MAGNETICA

ΔW_e VARIAZIONE DI ENERGIA ELETTRICA

$W = W_m + W_e$ ENERGIA TOTALE

CONSEGUENZE

① LA FREQUENZA DI RISONANZA PUÒ ESSERE VARIATA PERTURBANDO LA CAVITÀ

② IL "SEGNO" DELLA VARIAZIONE DIPENDE DA

- AUMENTO / RIDUZIONE VOLUME
- CAMPO ELETTRICO / MAGNETICO SULLA PERTURBAZIONE

③ SE PERTURBO DOVE $\underline{E}, \underline{H} = 0 \Rightarrow \omega = \omega_0$

SE PERTURBO DOVE $\underline{E}, \underline{H}$ MAX $\Rightarrow \omega - \omega_0$ È MAX

MISURE DI CAMPO (METODO PALLINA)

SE $\Delta V << \lambda^3$

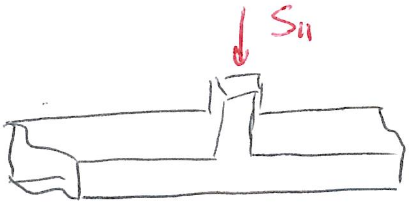
$$\frac{\Delta\omega}{\omega_0} = \frac{\Delta f}{f_0} = -\left(k_{E\parallel}\frac{|E_\parallel|^2}{W} + k_{E\perp}\frac{|E_\perp|^2}{W}\right) + \left(k_{H\perp}\frac{|H_\perp|^2}{W} + k_{H\parallel}\frac{|H_\parallel|^2}{W}\right)$$

OGGETTI METALLICI $\rightarrow$ E, H

" DIELETTRICI $\rightarrow$ E

POSSIBILE ISOLARE H CON 2 MISURE

PER GUIDE D'ONDA / STRUTTURE TW



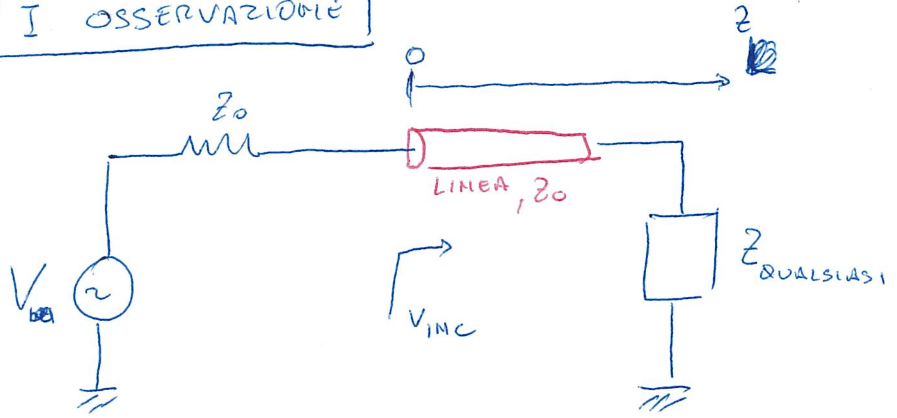
$$\Delta S_{11} = S_{11}^{P} - S_{11}^{UP}$$

$$2 P_i \, \Delta S_{11}(\omega) = -j\omega \left[k_E E^2 - k_H H^2 \right]$$

$$E^2 = |E|^2 e^{j 2 \phi_E}$$

LEGAME FRA LINEE DI TRASMISSIONE E CIRCUITI EQUIVALENTI

I OSSERVAZIONE



COEFF. DI RIFIESSIONE

V_{INC} TENSIONE INCIDENTE SULLA LINEA

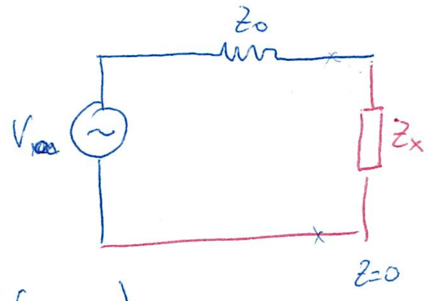
$V(z=0)$ TENSIONE MISURATA a $z=0$ $\quad V(z=0) = (1+\rho)\, V_{IN}$

V TENSIONE DEL GENERATORE

$\forall$ $Z_{QUALSIASI}$ LA TENSIONE INCIDENTE SULLA LINEA È SEMPRE PARI A $V_{IN}/2$ SE Z_0 È LA Z caratteristica della linea

DIMOSTRAZIONE

CIRCUITO EQUIVALENTE



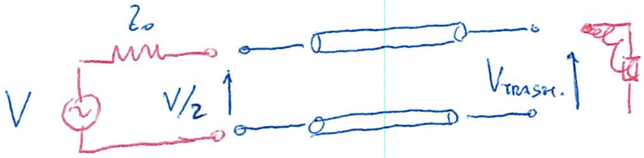
$$\rho = \frac{Z_x - Z_0}{Z_x + Z_0}$$

$$V(z=0) = V \frac{Z_x}{Z_x + Z_0} = V_{INC}(1+\rho)$$

$$= V_{INC} \frac{2 Z_x}{Z_x + Z_0} \Rightarrow 2 V_{INC} = V \qquad \boxed{V_{INC} = \frac{V}{2}}$$

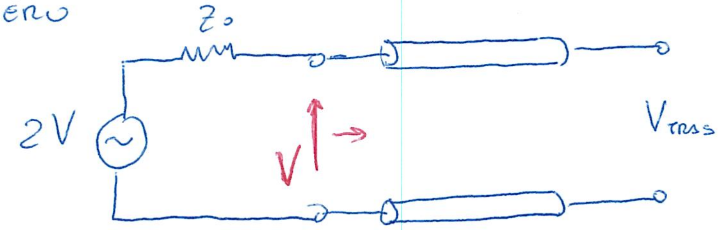
QUINDI

MISURA DI S_{21}:



$$S_{21} = \frac{V_{TRASM}}{V_{INC}} = \frac{2\, V_{TRASM}}{2\, V}$$

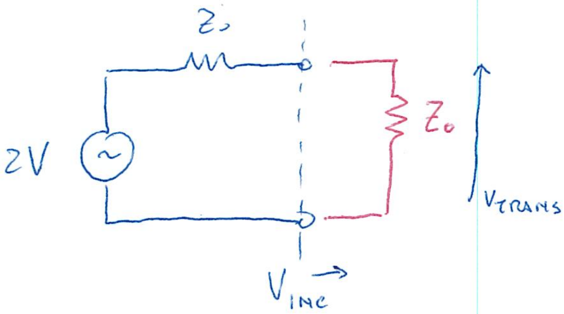
OVVERO



$$S_{12} = \frac{V_{TRAS}}{V}$$

CASO PRATICO

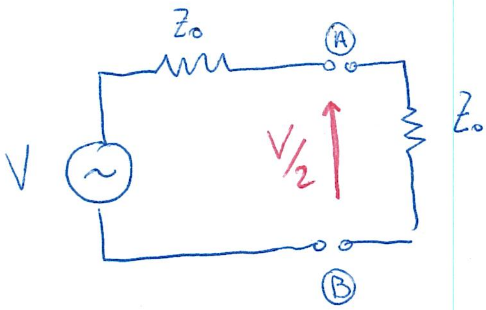
CARICO ADATTATO $\Rightarrow$ $S_{21} = 1$



$$V_{TRANS} = \frac{2V}{2}$$

$$S_{21} = \frac{V_{TRANS}}{V} = 1$$

OVVERO NELLA MANIERA SOLITA DI VEDERE LE COSE

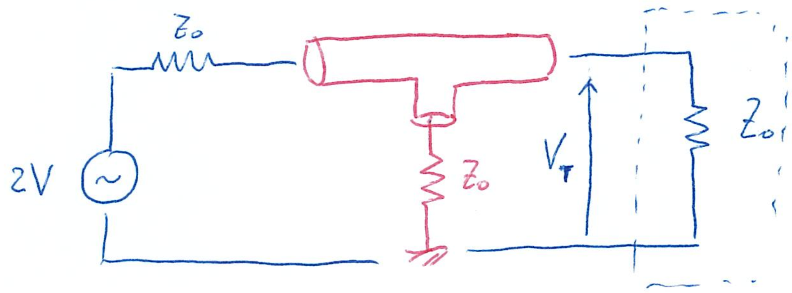


- LA TENSIONE SU Z_0 (V_{AB}) E' PROPRIO $V/2$ PERCHÈ NON
- C'È RIFLESSIONE

$$V_{INC} = V_{AB} = V/2$$

ESEMPI UTILI

GIUNZIONE A T: MISURE DI S_{21}



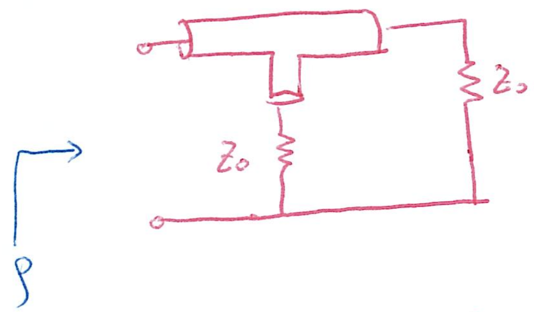
$$S_{21} = \frac{V_T}{V}$$

STRUMENTO CALIBRATO

$$V_T = 2V \, \frac{Z_0/2}{Z_0 + Z_0/2} = \frac{2}{3} V \qquad S_{21} = \frac{2}{3} = -3,522 \text{ dB}$$

PER COMPLETEZZA

GIUNZIONE A T CON DUE CARICHI DA 50 Ω

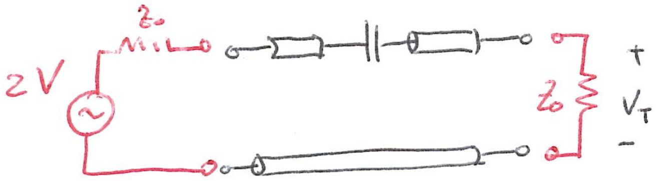


$$\rho = \frac{Z_0/2 - Z_0}{Z_0/2 + Z_0} = \frac{1}{3}$$

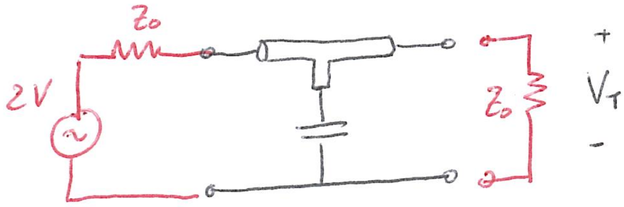
$$\Rightarrow \quad S_{11} = \frac{1}{3} = -9,542 \text{ dB}$$

S_{12} DI ELEMENTI REATTIVI

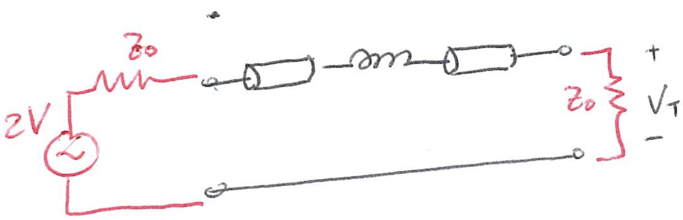
$$S_{12} = V_T / V$$



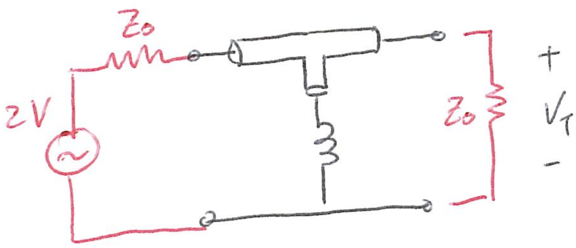
$$S_{12} = \frac{j\,\omega/\omega_0}{1 + j\,\omega/\omega_0} \qquad \omega_0 = \frac{1}{2 Z_0 C}$$



$$S_{12} = \frac{1}{1 + j\,\omega/\omega_0} \qquad \omega_0 = \frac{2}{Z_0 C}$$

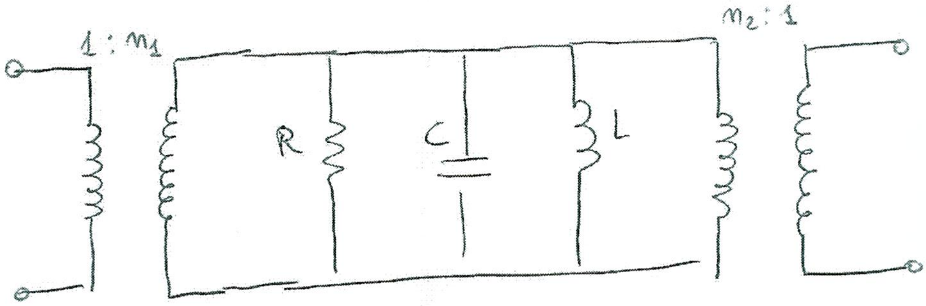


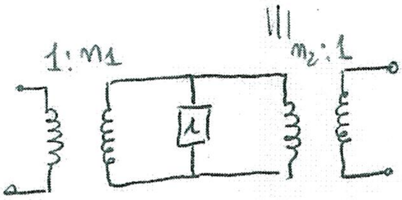
$$S_{12} = \frac{1}{1 + j\,\omega/\omega_0} \qquad \omega_0 = \frac{2 Z_0}{L}$$



$$S_{12} = \frac{j\,\omega/\omega_0}{1 + j\,\omega/\omega_0} \qquad \omega_0 = \frac{Z_0}{2L}$$

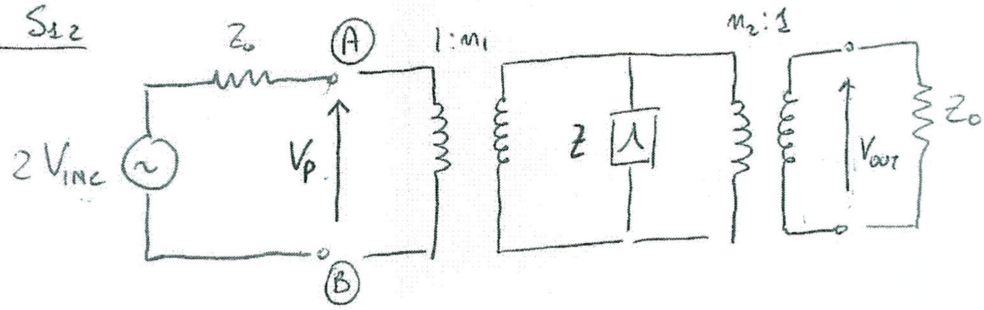
S_{12} CAVITÀ





$$z = \frac{R}{1 + jQ_0\delta}$$

CALCOLO S_{12}



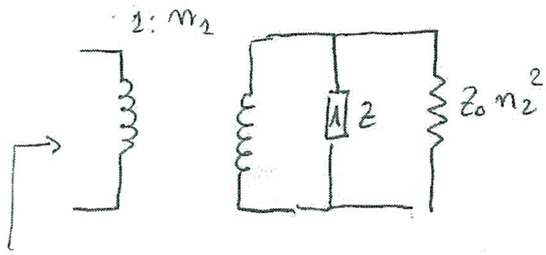
$V_p = V_{INC}(1+\rho)$ $\quad V_p = V_{INCIDENTE} + V_{RIFLESSA}$ $\quad V_{OUT} =$

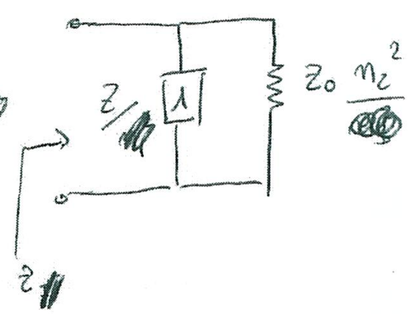
$$S_{21} = \frac{V_{OUT}}{V}$$

$$V_{OUT} = \frac{V_p\, n_1}{n_2} = V_{INC}(1+\rho)\frac{n_1}{n_2}$$

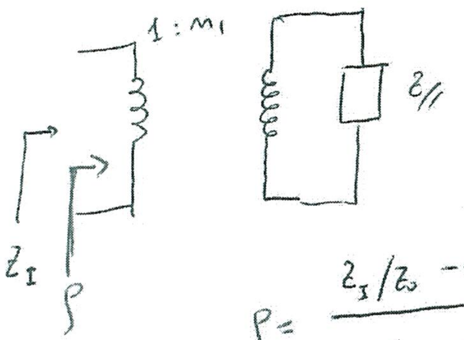
$$S_{12} = \frac{V_{OUT}}{V_{INC}} = (1+\rho)\frac{n_1}{n_2}$$

ρ COEF. DI RIFLESSIONE @ A-B





$$Z_{\parallel} = \frac{\frac{R}{1+jQ_0\delta}\cdot Z_0 n_2^2}{\frac{R}{1+jQ_0\delta} + Z_0 n_2^2} = \frac{R}{1+\beta_2+jQ_0\delta} \qquad \text{con } \frac{R}{n_2^2} = \beta_2 Z_0$$



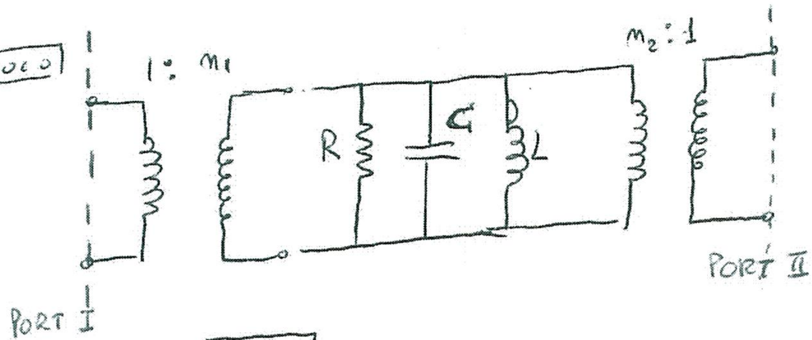
$$Z_I = \frac{R/n_1^2}{1+\beta_2+jQ_0\delta} = \frac{\beta_1 Z_0}{1+\beta_2+jQ_0\delta}$$

$$\rho = \frac{Z_I/Z_0 - 1}{Z_I/Z_0 + 1} = \frac{\beta_1 - 1 - \beta_2 - jQ_0\delta}{1+\beta_1+\beta_2+jQ_0\delta}$$

$$1+\rho = \frac{2\,Z_I/Z_0}{Z_I/Z_0+1} = \frac{2\beta_1}{1+\beta_1+\beta_2+jQ_0\delta}$$

$$S_{12} = (1+\rho)\frac{n_1}{n_2} = \boxed{\frac{2\sqrt{\beta_1\beta_2}}{1+\beta_1+\beta_2+jQ_0\delta}} \qquad \left(\beta_1\frac{n_1}{n_2} = \frac{R/Z_0}{n_1 n_2} = \sqrt{\beta_1\beta_2}\right)$$

RICAPITOLO



$$S_{12} = \frac{2\sqrt{\beta_1\beta_2}}{1+\beta_1+\beta_2+jQ_0\delta}$$

$$S_{11} = \frac{\beta_1 - 1 - \beta_2 - jQ_0\delta}{1+\beta_1+\beta_2+jQ_0\delta} \quad \text{con port 2 @ } Z_0$$

$$S_{22} = \frac{\beta_2 - 1 - \beta_1 - jQ_0\delta}{1+\beta_1+\beta_2+jQ_0\delta} \quad \text{con port 1 @ } Z_0$$

CON L'ALTRA PORTA APERTA

$$S_{11} = \frac{\beta_1 - 1 - jQ_0\delta}{\beta_1 + 1 + jQ_0\delta}$$

$$S_{22} = \frac{\beta_2 - 1 - jQ_0\delta}{\beta_2 + 1 + jQ_0\delta}$$