5.10.2 Calculation of total measurement uncertainty

Error sources contributing to the total measurement uncertainty depend on the type of measurement. In the following sections, error sources encountered in frequency measurement applications are described.

Measurement of absolute level
If the absolute level of a sinusoidal signal is to be measured, the following factors usually contribute to the total measurement uncertainty:

- Absolute level error

- Frequency response
  (only if the signal frequency distinctly differs from the frequency of the internal calibration source)

- Attenuator error
  (only if the attenuator setting differs from that specified in the data sheet for the absolute error)

- IF gain error
  (only if the set reference level differs from that specified in the data sheet for the absolute error)

- Linearity error
  The linearity error to be taken into account depends on the spacing of the input signal from the reference level.

- Bandwidth switching error
  (only if the set bandwidth differs from that specified in the data sheet for the absolute error)

An additional bandwidth error has to be taken into account in noise or channel power measurements.

Relative level measurement
The following error contributions have to be taken into account when measuring the level difference between two sinusoidal signals.

- Frequency response
  (only if the signal frequency strongly varies between the individual measurements)

- Attenuator error
  If the attenuator setting is not varied during measurement, this error can be ignored.

- IF gain error
  If the reference level is not varied during measurement, this error can be ignored.

- Linearity error

- Bandwidth switching error
  If the bandwidth is not varied during measurement, this error can be ignored.

An additional bandwidth error has to be taken into account in noise or channel power measurements if the resolution bandwidth is varied between the measurements.

Resolution bandwidth, attenuator setting (RF attenuation) and reference level should not be varied during the measurement in order to minimize the error of relative level measurement. Only the linearity error and frequency response, if applicable, will then contribute to the total measurement uncertainty.

Table 5-2 shows the error contributions that have to be considered in a number of typical measurements. The maximum error (worst case error) can be calculated from the individual contributions simply by adding the relevant parameters. The calculated maximum error has a confidence level of 100% so that the actual error of a measurement never exceeds the calculated error limits.

In practice, however, the maximum error seldom occurs. If the total uncertainty is the sum of many individual errors stemming from completely independent sources, it is statistically a very rare event that all individual errors occur in a measurement simultaniously at their maximum value and same sign.

Measurement / Error contribution	Absolute level of CW signal	Harmonic distortion	3rd order intermodulation products (close to carrier)	3rd order intercept	Channel power	Adjacent-channel power ratio	Power versus time (e.g. for TDMA signals), relative	Phase noise, far off carrier, with variation of RF attenuation and reference level	Phase noise, close to carrier
Absolute error	●			●	●				
Frequency response	●	●		●	●				
Attenuator error	●			●	●			●	
IF gain error	●			●	●			●	
Linearity error	●	●	●	●	●	●	●	●	●
Bandwidth switching error	●			●	●				
Bandwidth error					●	●		●	●
Error due to limited number of samples					●	●			
Mismatch error	●	●		●	●				

Table 5-2 *Error contributions in typical measurements using a spectrum analyzer*

It is more realistic to calculate the total uncertainty with a certain confidence level, which is typically 95% or 99%. Such calculation is permissible if the total uncertainty is composed of several error contributions of equal magnitude.

The distribution of the individual error contributions depends on the type of error. The following discussions are based on [5-5].

For random errors, that is for all errors listed above with the exception of mismatch errors, a rectangular distribution is assumed. The variance σ^2 of the individual errors is given by:

$$\sigma^2 = \frac{a^2}{3} \qquad \textit{(Equation 5-49)}$$

where σ^2 = variance
a = max. systematic error, in dB

If the data sheet specification of the level error is not given as worst case but with a certain confidence level, the variance has to be calculated from this value first. The following applies:

$$\sigma^2 = \left(\frac{a_{CL}}{k}\right)^2 \qquad \textit{(Equation 5-50)}$$

where σ^2 = variance
a_{CL} = specified error with certain confidence level or standard uncertainty, in dB

The value of k depends on the confidence level of the value specified in the data sheet. The following applies:

$$k = \sqrt{2} \cdot erfinv\left(\frac{CL}{100}\right) \qquad \textit{(Equation 5-51)}$$

where erfinv = inverse error function
CL = confidence level, in %

Fig. 5-27 shows k as a function of the confidence level. For a confidence level of 95%, k assumes a value of 1.96, and for 99%, a value of 2.58.

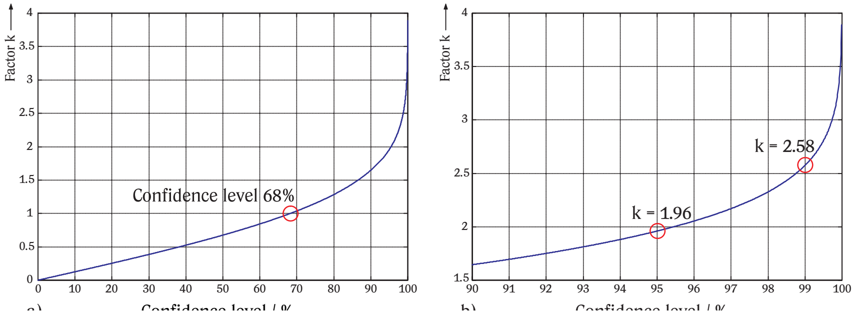

***Fig. 5-27** Coverage factor k as a function of confidence level*
a) confidence level 0% to 100%, b) confidence level 90% to 100% (zoomed)

In some cases the standard uncertainty s is specified in addition to the level error. This makes the calculation according to equation 5-50 unnecessary. The variance can be calculated from the specified standard uncertainty simply by squaring it.

Bandwidth errors are usually specified as a percentage. The following applies:

$$\sigma^2 = \frac{\left\{10 \cdot \log\left(1 + \frac{\Delta_{RBW} / \%}{100}\right)\right\}^2}{3} \qquad \textbf{(Equation 5-52)}$$

where σ^2 = variance
Δ_{RBW} = bandwidth error, in %

Errors due to mismatch have a U-shape distribution. The variance σ^2 is given by

$$\sigma^2 = \frac{\{20 \cdot \log(1 - r_s \cdot r_l)\}^2}{2} = \frac{\left\{20 \cdot \log\left(1 - \frac{s_s - 1}{s_s + 1} \cdot \frac{s_l - 1}{s_l + 1}\right)\right\}^2}{2} \qquad \textbf{(Equation 5-53)}$$

where σ^2 = variance
r_s = magnitude of source reflection coefficient
r_l = magnitude of spectrum analyzer reflection coefficient
s_s = VSWR of source
s_l = VSWR of spectrum analyzer

The reflection coefficient can be calculated from Equation 5-47.

Error	Calculation of variance
Absolute level error	$\sigma^2 = \frac{a^2}{3}$ and *Equation 5-49* ***and*** *Equation 5-50*
Frequency response	
Attenuator error	
IF gain error	
Linearity error	
Bandwidth switching error	
Bandwidth error	$\sigma^2 = \frac{\left\{10 \cdot \log\left(1 + \frac{\Delta_{RBW} / \%}{100}\right)\right\}^2}{3}$ *Equation 5-52*
Mismatch error	$\sigma^2 = \frac{\{20 \cdot \log(1 - r_s \cdot r_l)\}^2}{2}$ *Equation 5-53*

Table 5-3 *Calculating the variance of specified error contributions*

The combined standard uncertainty σ_{tot} can be calculated from the variances σ_i^2 of the individual contributions as follows:

$$\sigma_{tot} = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2} \qquad \text{(Equation 5-54)}$$

It has a confidence level of 68% (Fig. 5-27a). To maintain the error at some other confidence level, σ_{tot} tot has to be multiplied by a factor k which can be derived from Fig. 5.27. For a confidence level of 95%, k = 1.96 and for 99%, k = 2.58.

Example:
For the absolute level measurement of a sinusoidal input signal of 1 GHz (output VSWR of signal source 1.2:1), the total measurement uncertainty is to be determined with a confidence level of 95%. The resolution bandwidth set on the spectrum analyzer is 30 kHz, the RF attenuation is 20 dB and the reference level 0 dBm. The signal level is about 20 dB below the reference level.

Which errors contribute to the total measurement uncertainty?

- Absolute error
- Frequency response
- Attenuator error
- IF gain error
- Linearity error
- Bandwidth switching error

Since the input is a sinusoidal signal, the bandwidth error does not affect the total measurement uncertainty.

The required data are taken from the spectrum analyzer data sheet:

	Specified error	**Variance σ_i^2**
Absolute error	0.2 dB	$13.3 \cdot 10^{-3}$
Frequency response	0.5 dB	$83.3 \cdot 10^{-3}$
Attenuator error	0.2 dB	$13.3 \cdot 10^{-3}$
IF gain error	0.2 dB	$13.3 \cdot 10^{-3}$
Linearity error	0.2 dB	$13.3 \cdot 10^{-3}$
Bandwidth switching error	0.1 dB	$3.3 \cdot 10^{-3}$
Mismatch error		
VSWR at spectrum analyzer input	1.5	
VSWR at signal source output	1.2	$12.7 \cdot 10^{-3}$

The combined standard uncertainty can be calculated from the variances σ_i^2 with the aid of Equation 5-54 to yield $\sigma_{tot} = 0.39$. The total measurement error of 0.76 dB is obtained at a confidence level of 95% by multiplying the standard uncertainty by a factor of 1.96.

To simplify such error calculations, a spreadsheet in MS Excel® 5.0 is available (file FSP_ERR.XLS, Fig. 5-28) which can be obtained via the R&S Web site (www.rohde-schwarz.com).

Error Calculation for Rohde & Schwarz Spectrum Analyzers					
Inherent errors	unit	s = stand. uncertainty w = worst case	specified error	variance σ_i^2	contribute y = yes , n = no
		(1)	(2)	(3)	(4)
Absolute error 120 MHz	dB	w	0.3	0.03	y
Frequency response	dB	w	0.2	0.01	y
Input attenuator	dB	w	0.2	0.01	y
If gain	dB	w	0.2	0.01	y
Log linearity	dB	w	0.2	0.01	y
Bandwidth switching error	dB	w	0.2	0.01	y
Bandwidth error	%		10.00	0.07	y
Combined variance		$\sigma_{tot}^2 = \sigma_1^2 + \sigma_2^2 + ... + \sigma_n^2$		0.17	
Combined standard uncertainty		$\sigma_{tot} = \sqrt{\sigma_{tot}^2}$		0.41	
Total error (95% confidence level)	dB			0.80	(5)
(99% confidence level)	dB			1.05	
Error due to source mismatch		a = return loss / dB v = VSWR	specified values		
VSWR of SA		(6) v	(7) 3.1		
VSWR of DUT		v	1.57	0.55	
Combined variance		$\sigma_{tot}^2 = \sigma_1^2 + \sigma_2^2 + ... + \sigma_n^2$		0.71	
Combined standard uncertainty		$\sigma_{tot} = \sqrt{\sigma_{tot}^2}$		0.85	
Error including source mismatch (95%)	dB			1.66	(8)
(99%)	dB			2.18	

Fig. 5-28 *Spreadsheet FSP_ERR.XLS*

Legend for spreadsheet FSP_ERR.XLS

All input fields in the table are highlighted yellow. The fields for intermediate results and the resulting total level are highlighted in light and dark blue.

1 You can choose whether the values entered in (2) are absolute error limits (worst case) or a standard uncertainty.
2 Input fields for specified errors.
3 Output of the variances calculated from the input values.
4 You can choose whether the error entered under (2) is to be taken into account in calculating the total error. Error contributions can thus very easily be ignored without having to set the value entered under (2) to zero.
5 Output of the calculated total error with a confidence level of 95% or 99%. Errors due to mismatch are not considered in this result.
6 You can choose whether the mismatch of the DUT or spectrum analyzer is entered as VSWR (v) or as return loss (a).
7 Input field for specified maximum mismatch of the DUT or spectrum analyzer.
8 Output of the calculated total error with a confidence level of 95% or 99%. All error sources are considered in the result.