Measuring Parasitic Capacitance and Inductance Using TDR

Time-domain reflectometry (TDR) is commonly used as a convenient method of determining the characteristic impedance of a transmission line or quantifying reflections caused by discontinuities along or at the termination of a transmission line. TDR can also be used to measure quantities such as the input capacitance of a voltage probe, the inductance of a jumper wire, the end-to-end capacitance of a resistor, or the effective loading of a PCI card. Element values can be calculated directly from the integral of the reflected or transmitted waveform.

by David J. Dascher

Why would anyone use TDR to measure an inductance or capacitance when there are many inductance-capacitance-resistance (LCR) meters available that have excellent resolution and are easy to use? First of all, TDR allows measurements to be made on devices or structures as they reside in the circuit. When measuring parasitic quantities, the physical surroundings of a device may have a dominant effect on the quantity that is being measured. If the measurement cannot be made on the device as it resides in the circuit, then the measurement may be invalid. Also, when measuring the effects of devices or structures in systems containing transmission lines, TDR allows the user to separate the characteristics of the transmission lines from the characteristics of the device or structure being measured without physically separating anything in the circuit. To illustrate a case where TDR can directly measure a quantity that is very difficult to measure with an LCR meter, consider the following example.

A printed circuit board has a long, narrow trace over a ground plane, which forms a microstrip transmission line. At some point, the trace goes from the top of the printed circuit board, through a via, to the bottom of the printed circuit board and continues on. The ground plane has a small opening where the via passes through it. Assuming that the via adds capacitance to ground, a model of this structure would be a discrete capacitance to ground between the top and bottom transmission lines. For now, assume that the characteristics of the transmission lines are known and all that needs to be measured is the value of capacitance to ground between the two transmission lines.

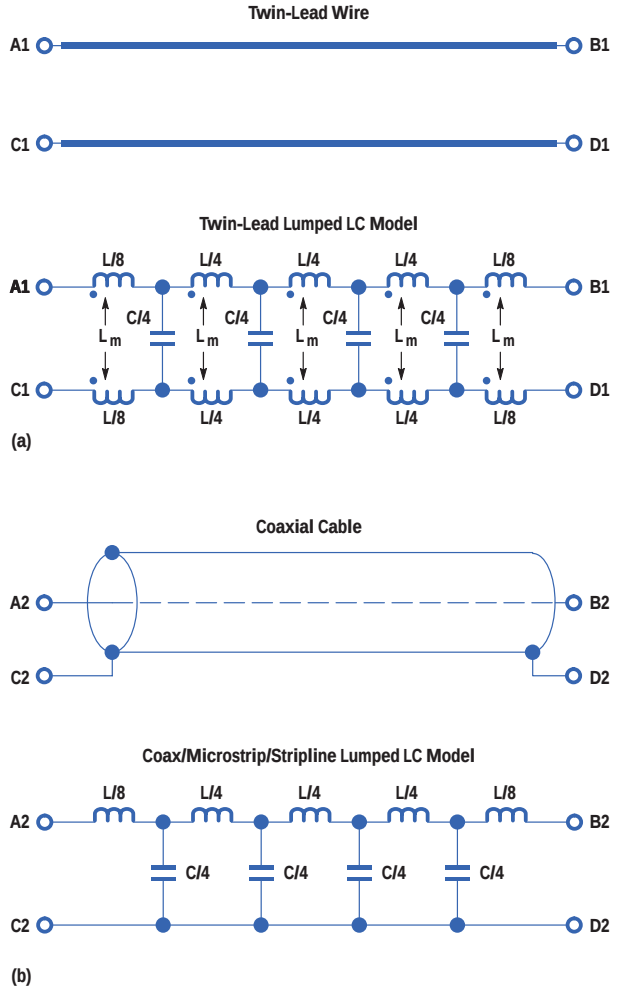
Using an LCR meter, the total capacitance between the trace-via-trace structure and ground can be measured but the capacitance of the via cannot be separated from the capacitance of the traces. To isolate the via from the traces, the traces are removed from the board. Now the capacitance between just the via and ground can be measured. Unfortunately, the measured value is not the correct value of capacitance for the model.

Using TDR instead of an LCR meter, a step-shaped wave is sent down the trace on the printed circuit board and the wave that gets reflected from the discontinuity caused by the via is observed. The amount of “excess” capacitance caused by the via can be calculated by integrating and scaling the reflected waveform. Using this method, the measured value of capacitance is the correct value of capacitance to be used in the model.

The discrepancy between the two measurements exists because the LCR meter was used to measure the total capacitance of the via while TDR was used to measure the excess capacitance of the via. If the series inductance of the via were zero, then its total capacitance would be the same as its excess capacitance. Since the series inductance of the via is not zero, a complete model of the via must include both its series inductance and its shunt capacitance. Assuming that the via is capacitive, the complete model can be simplified accurately by removing the series inductance and including only the excess capacitance rather than the total capacitance.

It should be no surprise that the value of excess capacitance measured using TDR is the correct value for the model. The reason to model the trace-via-trace structure in the first place is to predict what effect the via will have on signals propagating along the traces. TDR propagates a signal along the trace to make the measurement. In this sense, TDR provides a direct measurement of the unknown quantity.

Fig. 1. Two-conductor transmission lines and lumped LC models.



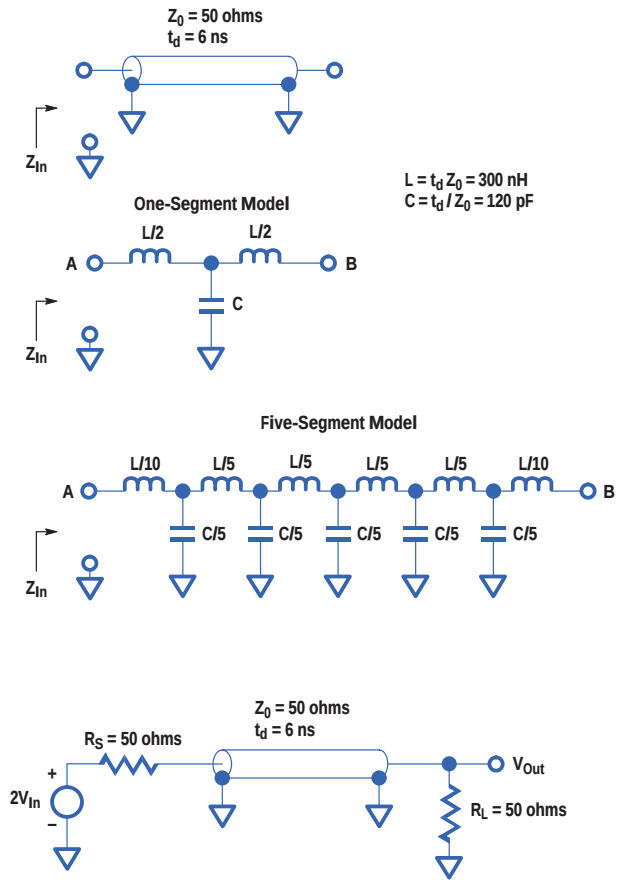
To derive expressions that relate TDR waveforms to excess capacitance and inductance, an understanding of fundamental transmission line parameters is required. This article presents a cursory review of transmission lines and the use of TDR to characterize them, and then derives expressions for excess capacitance and inductance. If you are already familiar with transmission lines and TDR, you may wish to skip the review sections.

Fundamental Transmission Line Parameters

First, a few words about "ground." Twin-lead (or twisted-pair) wire forms a two-conductor transmission line structure that can be modeled as shown in Fig. 1. The model includes the self-inductance of each conductor, mutual inductance between the self-inductances, and capacitance between the two conductors. Skin effect and dielectric loss are assumed to be negligible in this model. Injecting a current transient i into one side of the transmission line, from node A1 to node C1, causes a voltage $v = iZ_0$ to appear between nodes A1 and C1 and also causes a voltage $v = Ldi/dt$ to appear across the series inductance of both the A-B and C-D conductors. Referring back to the physical structure, this means that there is a voltage difference between the left side and the right side of each conductor. Even if you name the C-D conductor "ground," it still develops a voltage between its left side and its right side, across its series inductance.

Microstrip traces and coaxial cables (coax) are two special cases of two-conductor transmission line structures. Injecting a current transient into one side of a microstrip or coax transmission line causes a voltage to appear across only one of the two conductors. In the case of an ideal microstrip, where one of the conductors is infinitely wide, the wide conductor can be thought of as a conductor with zero inductance. Hence the voltage generated across the infinitely wide conductor is zero. With coax, the inductance of the outer conductor is not zero. However, the voltage generated across the inductance of the outer conductor has two components. One component is a result of the current through the self-inductance of the outer conductor ($v_1 = L_{oc}di/dt$). The other component is a result of the current through the center conductor and the mutual inductance between the center and outer conductors ($v_2 = L_m di/dt$).

Fig. 2. *Two LC models for a coaxial cable. The five-segment model is accurate over a wider range of frequencies than the one-segment model.*



Current that enters the center conductor returns through the outer conductor, so the two currents are equal but in opposite directions. The unique property of coax is that the self-inductance of the outer conductor is exactly equal to the mutual inductance between the center and the outer conductors. Hence the two components that contribute to the voltage generated across the inductance of the outer conductor exactly cancel each other and the resulting voltage is zero. When current is injected into a coax transmission line, no voltage is generated along the outer conductor if the current in the center conductor is returned in the outer conductor.

The point here is that the generalized model for a two-conductor transmission line can be simplified for microstrip and coax constructions. The simplified model has zero inductance in series with one of the conductors since, in both cases, no voltage appears across the conductor. This conductor is commonly referred to as the ground plane in microstrip transmission lines and as the shield in coax transmission lines. The other conductor, the one that develops a voltage across it, is referred to as the transmission line, even though it is really only half of the structure.

There are two ways to model a lossless transmission line. One method defines the transmission line in terms of characteristic impedance (Z_0) and time delay (t_d) and the other method defines the transmission line in terms of total series inductance (L_{Total}) and total shunt capacitance (C_{Total}). There are times when it is beneficial to think of a transmission line in terms of Z_0 and t_d and likewise, there are times when thinking in terms of C_{Total} and L_{Total} is best.

If one end of a long transmission line is driven with an ideal current step, the voltage at that end will be an ideal voltage step whose height is proportional to the current and the characteristic impedance Z_0 of the transmission line: $V_{In} = I_{In}Z_0$. The waveform generated at that end will propagate along the transmission line and arrive at the opposite end some time later. The time it takes the wave to propagate from one end to the other is the time delay (t_d) of the transmission line.

The total capacitance and inductance of a transmission line can be measured with an LCR meter. To determine the total capacitance of a coaxial cable, measure the capacitance between the center conductor and the shield at one end of the cable while the other end is left open. The frequency of the test signal used by the LCR meter should be much less than $1/(4t_d)$ where t_d is the time delay of the cable. To measure the total inductance of the cable, measure the inductance from the center conductor to the shield at one end of the cable while the other end of the cable has the center conductor connected to the shield. Again, the test frequency must be much less than $1/(4t_d)$.

If the characteristic impedance and time delay of a transmission line are known, then the total shunt capacitance and total series inductance of the transmission line can be calculated as:

$$C_{Total} = \frac{t_d}{Z_0}$$

$$L_{Total} = t_d Z_0 .$$

If the total shunt capacitance and total series inductance are known, then Z_0 and t_d can be calculated as:

$$Z_0 = \sqrt{\frac{L_{Total}}{C_{Total}}}$$

$$t_d = \sqrt{L_{Total} C_{Total}} .$$

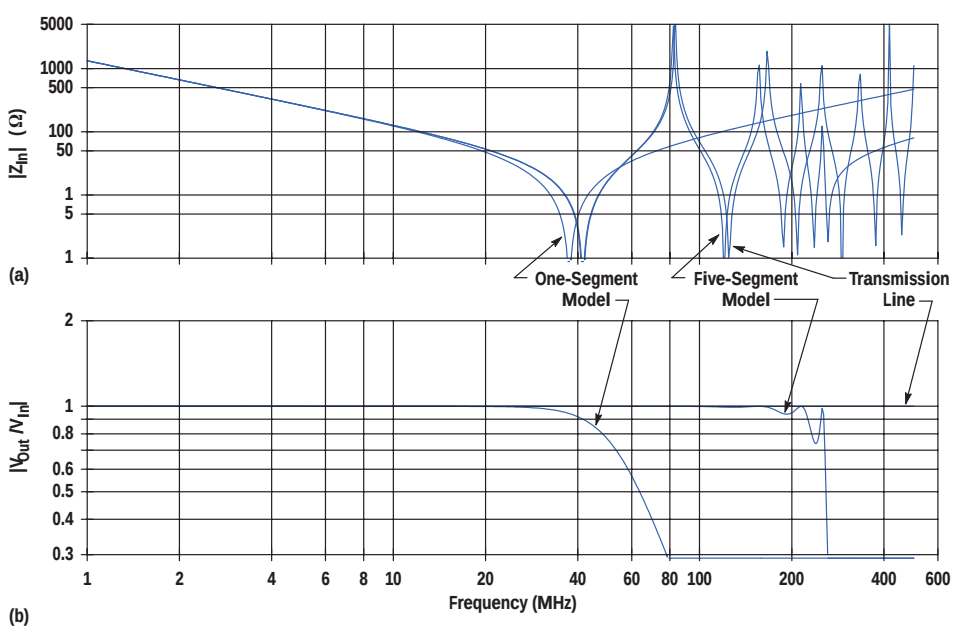
As an example, many engineers use 50-ohm coaxial cables that are about four feet long and have BNC connectors on each end. The time delay of these cables is about six nanoseconds, the capacitance is 6 ns/50 ohms = 120 picofarads, and the inductance is 6 ns × 50 ohms = 300 nanohenrys.

Fig. 2 shows two LC models for a 50-ohm, 6-ns-long coaxial cable. The distributed inductance of the transmission line has been collected into two discrete inductors in the first model and six discrete inductors in the second model. The distributed capacitance has been collected into one discrete capacitor in the first model and five discrete capacitors in the second. Collecting the distributed capacitance and inductance into discrete, lumped elements reduces the range of frequencies over which the model is accurate. The more discrete segments there are, the wider the range of frequencies over which the model is accurate. Fig. 3 shows the magnitude of the impedance seen looking into one end of each model while the other end is left open. The five-segment model is accurate over a wider range of frequencies than the one-segment model.

Fig. 3 also shows the transmitted response through each of the models in a circuit that is both source and load terminated in 50 ohms. The discrete LC models are both low-pass filters. Again, the five-segment model is accurate over a wider range of frequencies than the one-segment model. In the time domain, a good rule of thumb is to break a transmission line into five segments per rise time. For example, to build a model that will accurately simulate the response of a step with a 1-ns rise time, the model should contain five segments per nanosecond of time delay. This would require a 30-segment model for the 4-foot, 6-ns coaxial cable.

The transmission line model used in SPICE and many other time-domain simulators is defined by characteristic impedance and time delay. It consists only of resistors, dependent sources, and time delay. It is an accurate model of a lossless transmission line over an infinite range of frequencies. There are three situations in which lumped LC models may be preferred over the Z_0-t_d model used by SPICE. When modeling very short sections of transmission lines, the maximum time increment is never greater than the time delay of the shortest transmission line. Short transmission

Fig. 3. *(a) Magnitude of the input impedance Z_{In} for the transmission line and models of Fig. 2. (b) $|V_{Out}/V_{In}|$ for the transmission line and models of Fig. 2 (in the circuit at the bottom of Fig. 2).*



lines can cause lengthy simulations. When modeling skin effect and dielectric loss, discrete LC models can be modified to include these effects. And finally, discrete LC models can be used to model transmission line systems that contain more than two conductors.

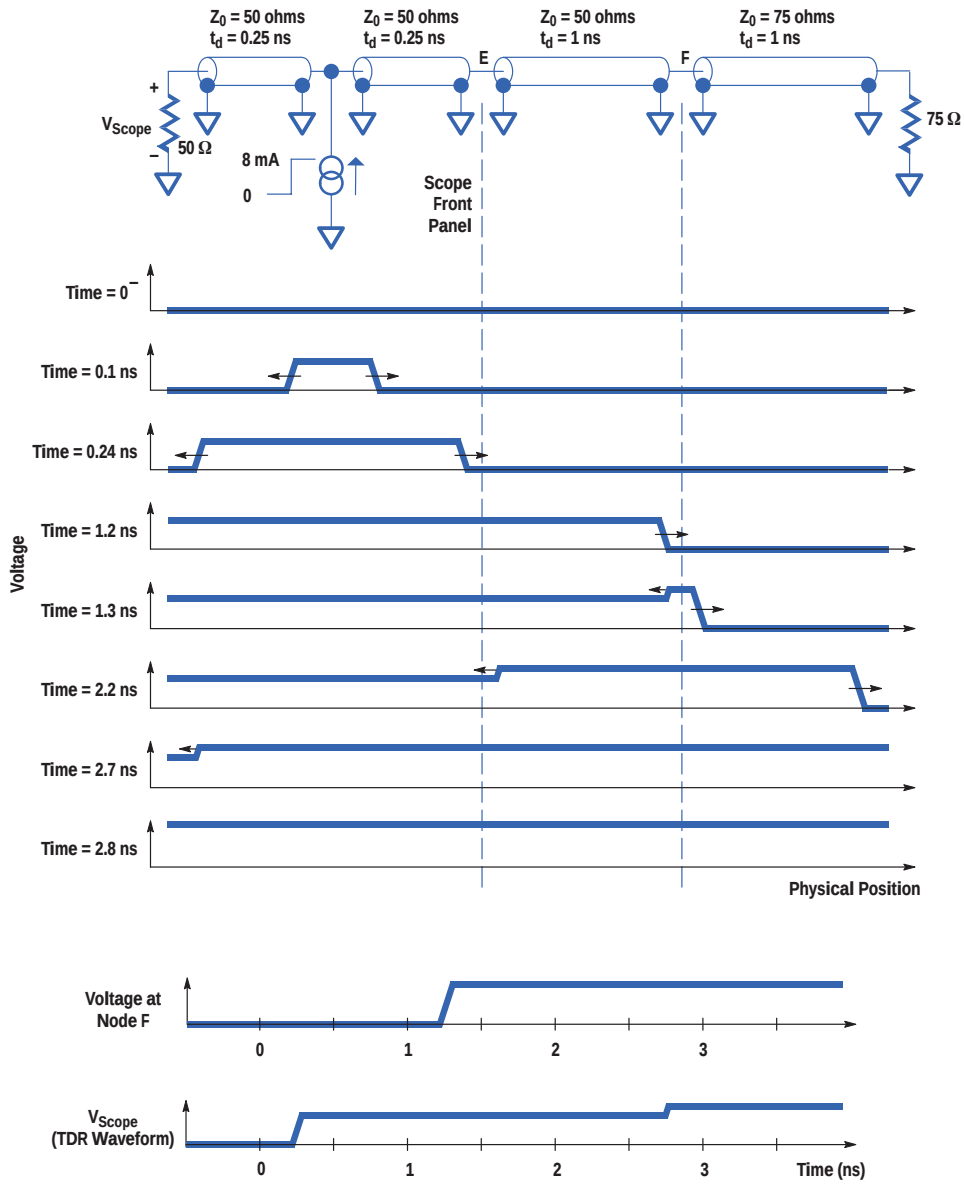
Characteristic impedance and time delay, or series inductance and shunt capacitance, completely specify the electrical properties of a lossless transmission line. Propagation velocity and length are also required to specify a physical cable or printed circuit board trace completely. If the time delay (also known as the electrical length) of a cable is 6 ns and the physical length of the cable is 4 feet, then the propagation velocity is 0.67 feet per nanosecond. The propagation delay is 1.5 ns per foot.

Reflection Coefficients for Impedance Changes

Fig. 4 shows a typical measurement setup using an oscilloscope with an internal TDR. The oscilloscope input is a 50-ohm transmission line terminated in 50 ohms. The TDR step generator can be modeled as a current source that connects to the 50-ohm transmission line between the front of the oscilloscope and the 50-ohm termination. When the current source transitions from low to high at time = 0, voltages are generated in the system as shown. Note that the upper traces in Fig. 4 are plots of voltage as a function of position along the transmission lines, not time. The waveform viewed by the oscilloscope is the waveform at the 50-ohm termination, which is shown at the bottom of Fig. 4. This is the TDR waveform.

Fig. 4. *Typical measurement setup and waveforms for an oscilloscope with an internal time-domain reflectometer (TDR).*

Click to View Animated Figure 4



There is only one discontinuity in the system: the transition between the 50-ohm cable and the 75-ohm cable at node F. When the wave propagating along the 50-ohm transmission line arrives at the 75-ohm transmission line, reflected and transmitted waves are generated. The wave that is incident on the transition is shown at time = 1.2 ns. Waves that are reflected and transmitted from the transition are shown at time = 1.3 ns. Defining a positive reflected wave to be propagating in the opposite direction from the incident wave, the reflected wave is given as:

$$\frac{\text{reflected}}{\text{incident}} = \frac{Z_B - Z_A}{Z_B + Z_A}.$$

where Z_A is the characteristic impedance through which the incident wave travels and Z_B is the characteristic impedance through which the transmitted wave travels. The transmitted wave is given as:

$$\frac{\text{transmitted}}{\text{incident}} = \frac{2Z_B}{Z_B + Z_A}.$$

If Z_A = 50 ohms, Z_B = 75 ohms, and the incident waveform is a 1V step, then the reflected waveform will be a step of height 1(75 – 50)/(75 + 50) = 0.2 volt, and the transmitted waveform will be a step of $1(2 \times 75)/(75 + 50) = 1.2$ volts.

It is important to note that the waveform that appears at node F is the waveform that is transmitted into the 75-ohm cable. The waveform at node F can be derived by simplifying the circuit at the transition. Looking back into the 50-ohm cable, the circuit can be modeled as a voltage source in series with a resistor. The source is twice the voltage of the step propagating along the 50-ohm cable and the resistor is the characteristic impedance of the 50-ohm cable. Looking into the 75-ohm cable, the circuit can be modeled as a 75-ohm resistor. The simplified circuit is just a voltage divider in which the divided voltage is $2V_{Step}Z_2/(Z_2 + Z_1) = 1.2V_{Step}$. This is the transmitted waveform.

The TDR waveform, the waveform at the 50-ohm termination of the oscilloscope, is shown at the bottom of Fig. 4. The incident step is shown at time = 0.25 ns and the reflected step is shown at time = 2.75 ns. The characteristic impedance of the 75-ohm transmission line can be calculated from the TDR waveform by solving for Z_B in the above relationship, which gives:

$$Z_B = Z_A\left(\frac{\text{incident} + \text{reflected}}{\text{incident} - \text{reflected}}\right).$$

Both the incident and reflected waveforms are step functions that differ only in amplitude and direction of propagation. In the expression for Z_B, the step functions cancel and all that remains is the step heights. The TDR waveform in Fig. 4 shows the incident step height to be +1V and the reflected step height to be +0.2V. Knowing that the impedance looking into the oscilloscope is Z_A = 50 ohms, the characteristic impedance of the second cable, Z_B, can be calculated as Z_B = 50(1V + 0.2V)/(1V – 0.2V) = 75 ohms.

There is a “transition” between the front panel of the oscilloscope and the first cable. The characteristic impedance on either side of the transition is the same, so the reflected wave has zero amplitude. If there is no reflected wave, the transmitted wave is identical to the incident wave. Electrically, there is no discontinuity at this transition since it causes no reflection and the transmitted wave is identical to the incident wave.

The transition between transmission lines of different characteristic impedances is a discontinuity that produces reflected and transmitted waves that are identical in shape to the incident wave. The reflected wave differs from the incident wave in amplitude and direction of propagation. The transmitted wave differs only in amplitude. Discontinuities that are not simply impedance changes will produce reflected and transmitted waves that are also different in shape from the incident wave.

Discrete Shunt Capacitance

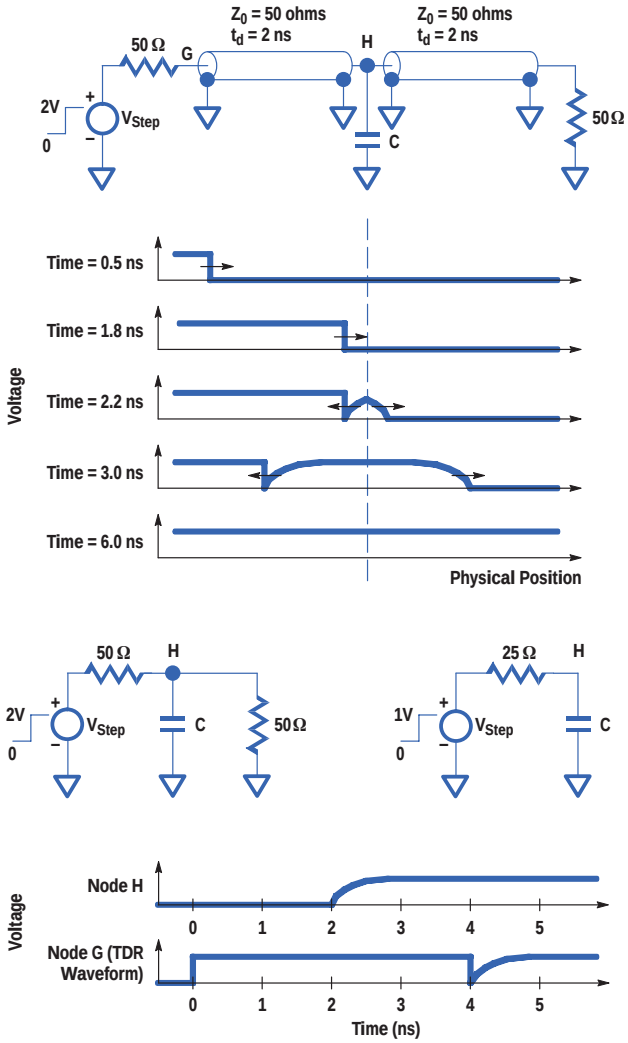
Fig. 5 shows two 50-ohm transmission lines with a shunt capacitor between them. The capacitance produces a discontinuity between the two transmission lines. A wave that is incident on the discontinuity produces reflected and transmitted waves. In this case, both the reflected and transmitted waves have a different shape than the incident wave.

Given an incident step, the voltage at node H can be ascertained by simplifying the circuit as shown in Fig. 5. The incident step, shown at time = 1.8 ns, has an amplitude of inc_ht and is propagating along a 50-ohm transmission line. Looking to the left of the capacitor, the circuit can be simplified as a voltage source in series with a resistor. The voltage source is twice the incident voltage and the resistance is Z_0 = 50 ohms (not R_S = 50 ohms). Looking to the right of the capacitor, the circuit can be simplified as a resistor of value Z_0 = 50 ohms (not R_L = 50 ohms). The simplified circuit contains no transmission lines. The circuit can again be simplified by combining the two resistors and the voltage source into their Thevenin equivalent.

Note that if the left transmission line did not have a source resistance of 50 ohms or the right transmission line did not have a load resistance of 50 ohms, the simplified model would still be valid for 4 ns after the incident step arrives at node H. After 4 ns, reflections from the source and load terminations would have to be accounted for at node H.

Fig. 5. Two 50-ohm transmission lines with a shunt capacitor between them. The capacitance C is to be measured.

Click to View Animated Figure 5



Looking at the simplified circuit and redefining time = 0 to be the time when the incident step arrives at node H, the voltage at node H is zero for time < 0. For time > 0, the voltage at node H is:

$$v_H = \text{inc_ht}\left(1 - e^{-\frac{t}{\tau}}\right),$$

where $\tau = RC = \frac{Z_0}{2}C$.

The voltage waveform that appears on node H is the transmitted waveform. The reflected wave can be determined knowing that the incident wave equals the transmitted wave plus the reflected wave (defining each wave as positive in its own direction of propagation). For time < 0, the reflected wave is zero. For time > 0, the reflected wave is:

$$\text{reflected} = -\text{inc_ht} \cdot e^{-\frac{t}{\tau}},$$

where $\tau = RC = \frac{Z_0}{2}C$.

Normalizing the reflected waveform to the incident step height and integrating the normalized reflected waveform gives:

$$\text{reflected_n} = \frac{\text{reflected}}{\text{inc_ht}}$$

$$\int_{-\infty}^{+\infty} \text{reflected_n} \cdot dt = \int_{0}^{+\infty} -e^{-\frac{t}{\tau}} dt = \tau e^{-\frac{t}{\tau}} \Bigg|_{0}^{+\infty} = -\tau.$$

Solving for the shunt capacitance between the two transmission lines gives:

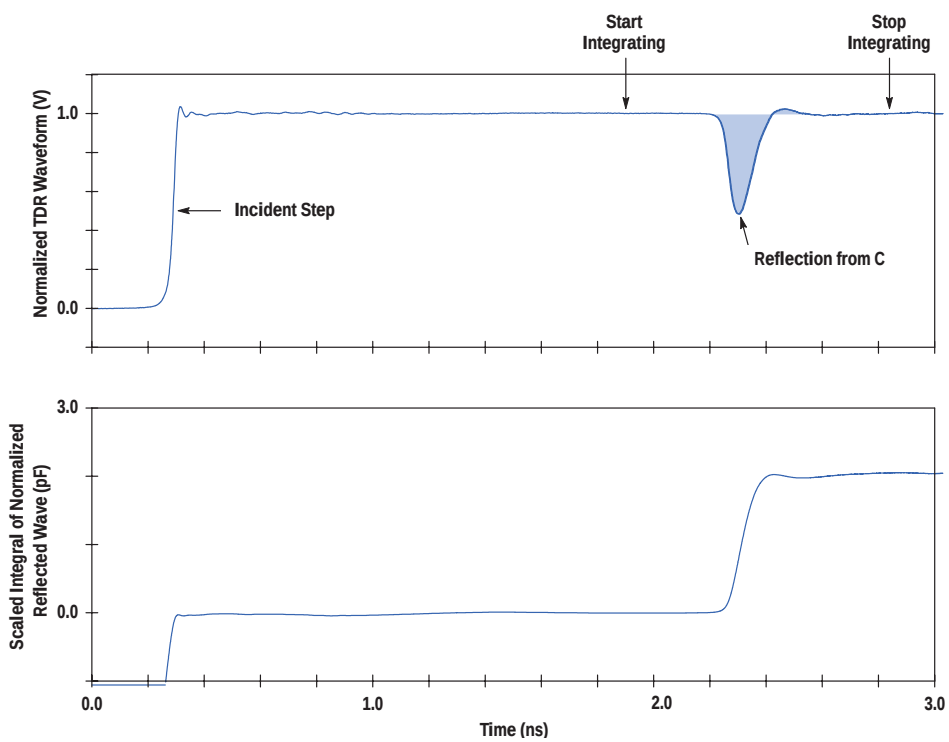
$$C = \frac{2\tau}{Z_0} = -\frac{2}{Z_0} \int_{0}^{+\infty} \text{reflected_n} \cdot dt.$$

Thus, the amount of capacitance between the two transmission lines can be determined by integrating and scaling the normalized reflected waveform.

Fig. 6 shows a measured TDR waveform from a 50-ohm microstrip trace with a discrete capacitor to ground at the middle of the trace. The TDR waveform has been normalized to have an incident step height of 1. The TDR waveform shows the incident step at 0.15 nanosecond and the reflected wave superimposed on the incident step at 2.3 nanoseconds. The reflected wave can be ascertained from the TDR waveform to be the TDR waveform minus one for all time after the incident step has settled. The bottom trace is the integral of the normalized reflected waveform scaled by $-2/Z_0 = -1/25$. Mathematically, the reflected waveform needs to be integrated until time = infinity, but practically, the reflected waveform only needs to be integrated until it has settled reasonably close to zero. In this case it takes about 500 picoseconds for the reflected wave to settle. The bottom trace shows that the discontinuity is a 2.0-pF capacitance.

Fig. 5 shows an ideal reflected wave to be an exponential with the same height as the incident step, but Fig. 6 shows a measured reflected wave to be similar to an exponential but with a rounded leading edge and a height much less than that of the the incident step. The measured reflected wave differs from the ideal reflected wave because of the nonzero rise time of the incident step and loss in the transmission lines. Neither of these nonideal conditions affects the integral of the reflected wave, as will be shown later.

Fig. 6. *(top) Measured TDR waveform from a 50-ohm microstrip trace with a discrete capacitor to ground at the middle of the trace. (bottom) The integral of the reflected waveform reveals that the capacitance is 2.0 pF.*



Discrete Series Inductance

Fig. 7 shows two 50-ohm transmission lines with a series inductor between them. Given an incident step, the circuit can be simplified as shown to determine the voltage waveforms at nodes J and K. The voltages at nodes J and K are zero for time < 0. For time > 0, the voltages at nodes J and K are:

$$v_J = \text{inc_ht}\left(1 + e^{-\frac{t}{\tau}}\right)$$

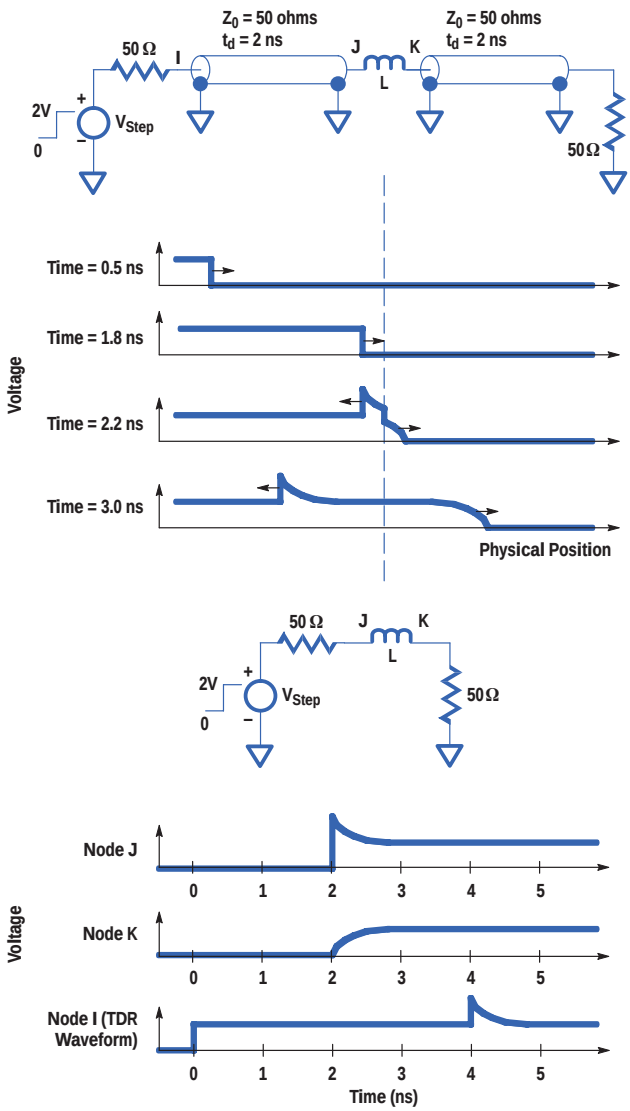
$$v_K = \text{inc_ht}\left(1 - e^{-\frac{t}{\tau}}\right)$$

where $\tau = \frac{L}{R} = \frac{L}{2Z_0}$.

In the previous case of a shunt capacitor, there was only one node between the two transmission lines at which the relationship incident = transmitted + reflected was applied. Now there are two nodes, each of which has unique incident, transmitted, and reflected waveforms. The voltage waveform at node J is the waveform that is transmitted to the inductor and the voltage waveform at node K is the waveform that is transmitted to the right transmission line. The waveform that is reflected at node J is the incident waveform minus the waveform transmitted to the inductor, that is, v_J.

Fig. 7. *Two 50-ohm transmission lines with a series inductor between them. The inductance L is to be measured.*

Click to View Animated Figure 7



For time < 0, the reflected wave is zero. For time > 0, the reflected wave is:

$$\text{reflected} = \text{inc_ht} \cdot e^{-\frac{t}{\tau}}.$$

Normalizing the reflected waveform to the incident step height and integrating the normalized reflected waveform gives:

$$\text{reflected_n} = \frac{\text{reflected}}{\text{inc_ht}}$$

$$\int_{-\infty}^{+\infty} \text{reflected_n} \cdot dt = \int_{0}^{+\infty} e^{-\frac{t}{\tau}} dt = -\tau e^{-\frac{t}{\tau}} \Bigg|_{0}^{+\infty} = \tau.$$

Solving for the series inductance between the two transmission lines gives:

$$L = 2Z_0\tau = 2Z_0 \int_{0}^{+\infty} \text{reflected_n} \cdot dt.$$

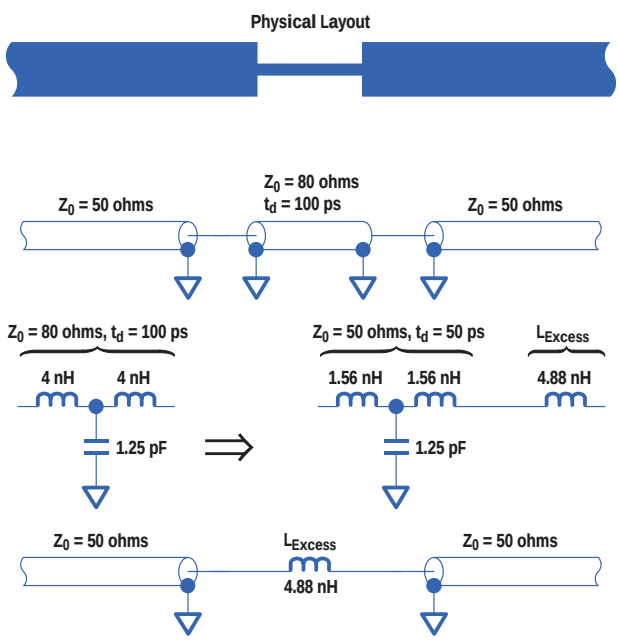
Thus, the amount of inductance between the two transmission lines can be determined by integrating and scaling the normalized reflected waveform.

Excess Series Inductance

In the previous two cases of shunt capacitance and series inductance, the discontinuity was caused by discrete, lumped capacitance or inductance. Discontinuities can also be caused by distributed shunt capacitance and series inductance. Consider the printed circuit board traces shown in Fig. 8. Being over a ground plane, the traces form microstrip transmission lines. The circuit can be modeled as two long, 1-ns, 50-ohm transmission lines separated by a short, 100-ps, 80-ohm transmission line. Modeling the discontinuity as a short, high-impedance transmission line produces accurate simulations up to very high frequencies, or for incident waveforms with very fast transitions.

The discontinuity can also be modeled as a single discrete inductor. Modeling the discontinuity as an inductor produces less-accurate simulations at high frequencies, but when transition times are much larger than the time delay of the discontinuity, a simple inductor produces accurate simulation results. The simpler model provides quick insight into the behavior of the circuit. To say a discontinuity looks like 4.9 nH of series inductance provides most people with

Fig. 8. *Two 50-ohm microstrip transmission lines with a distributed discontinuity between them. The discontinuity can be modeled as a short 80-ohm transmission line or (less accurately) as a series inductor. The correct inductance to be used to model the narrow trace is L_{Excess}. (If the discontinuity were much wider than the 50-ohm traces, instead of narrower as shown here, it could be modeled as a shunt capacitor.)*



more insight than to say it looks like a 100-ps-long, 80-ohm transmission line. Also, simulating a single inductor is much faster than simulating a very short transmission line.

The total series inductance of the 100-ps-long, 80-ohm transmission line is:

$$L_{Total} = t_d Z_{High} = 100 \text{ ps} \times 80 \text{ ohms} = 8 \text{ nH}.$$

To model the short trace as an 8-nH inductor would be incorrect because the trace also has shunt capacitance. The total shunt capacitance of the trace is:

$$C_{Total} = \frac{T_d}{Z_{High}} = \frac{100 \text{ ps}}{80 \text{ ohms}} = 1.25 \text{ pF}.$$

To simplify the model of the discontinuity, separate the total series inductance into two parts. One part is the amount of inductance that would combine with the total capacitance to make a section of 50-ohm transmission line, or "balance" the total capacitance. The remaining part is the amount of inductance that is in excess of the amount necessary to balance the total capacitance. Now, combine the total capacitance with the portion of the total inductance that balances that capacitance and make a short section of 50-ohm transmission line. Put this section of transmission line in series with the remaining, or excess, inductance. The short 50-ohm transmission line does nothing except increase the time delay between the source and load. The excess inductance (not the total inductance) causes an incident wave to generate a nonzero reflected wave and a nonidentical transmitted wave. This is the correct amount of inductance to use to model the short, narrow trace.

In this case, the portion of the total inductance that balances the total capacitance is:

$$L_{Balance} = C_{Total} Z_0^2 = 1.25 \text{ pF} \times (50 \text{ ohms})^2 = 3.125 \text{ nH}$$

and the remaining, excess inductance is:

$$\begin{aligned} L_{Excess} &= L_{Total} - L_{Balance} \\ &= 8 \text{ nH} - 3.125 \text{ nH} = 4.875 \text{ nH}. \end{aligned}$$

Expressing the excess inductance in terms of the time delay and impedance of the narrow trace (t_d, Z_{High}) and the impedance of the surrounding traces ($Z_0 = Z_{Ref}$) gives:

$$\begin{aligned} L_{Excess} &= L_{Total} - L_{Balance} \\ &= t_d Z_{High} - C_{Total} Z_{Ref}^2 = t_d Z_{High} - \frac{t_d}{Z_{High}} Z_{Ref}^2 \end{aligned}$$

$$L_{Excess} = t_d Z_{High} \left(1 - \left(\frac{Z_{Ref}}{Z_{High}} \right)^2 \right).$$

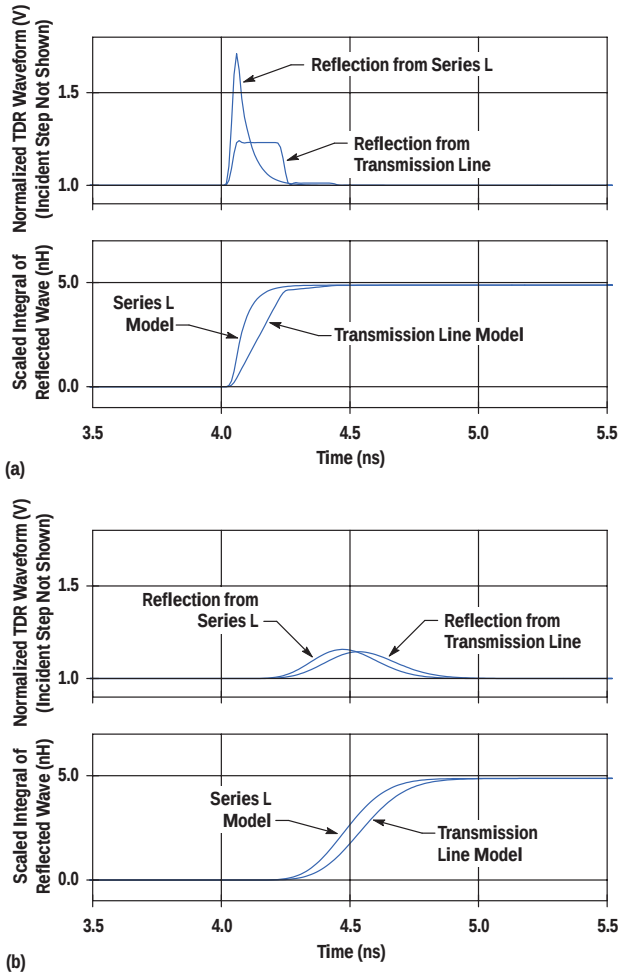
Fig. 9a shows TDR waveforms from a SPICE simulation that compare the reflection generated from an ideal 4.88-nH inductor to the reflection generated from an 80-ohm, 100-ps transmission line, both in a 50-ohm system. Fig. 9b shows TDR waveforms of the same circuit as Fig. 9a, but the rise time of the incident step in Fig. 9b is longer than the rise time used in Fig. 9a. Notice that when the rise time of the incident step is much larger than the time delay of the discontinuity, the simplified model agrees with the transmission line model. As the rise time of the incident step changes, the peak value of the reflected waveform also changes but the integral of the reflected waveform does not change. Using the integral rather than the peak value to quantify the discontinuities makes the measurement of excess inductance rise time independent.

Excess Shunt Capacitance

In the example shown in Fig. 8, the discontinuity is a section of trace that is narrower than the surrounding traces. This discontinuity can be modeled as an inductor. If a discontinuity is a section of trace that is wider than the surrounding traces, the discontinuity can be modeled as a capacitor. As in the previous case, the correct value of capacitance is not the total capacitance of the wide section of trace but rather the difference between the total capacitance and the portion of that capacitance that combines with the total inductance to make a section of transmission line of the same impedance as the surrounding, reference impedance. In terms of the time delay and impedance of the wide trace (t_d, Z_{Low}) and the impedance of the surrounding traces ($Z_0 = Z_{Ref}$), the excess capacitance is:

$$C_{Excess} = \frac{t_d}{Z_{Ref}} \left(1 - \left(\frac{Z_{Low}}{Z_{Ref}} \right)^2 \right).$$

Fig. 9. *(a) TDR waveforms from a SPICE simulation showing reflections from an ideal 4.88-nH inductor and an 80-ohm, 100-ps transmission line, both in a 50-ohm system. The final value of the integral of the reflected wave is the same for both. (b) Same as (a) for an incident step with a longer rise time. The agreement between the reflected waves for the single-inductor model and the transmission line is better. The final value of the integral of the reflected wave is unchanged.*



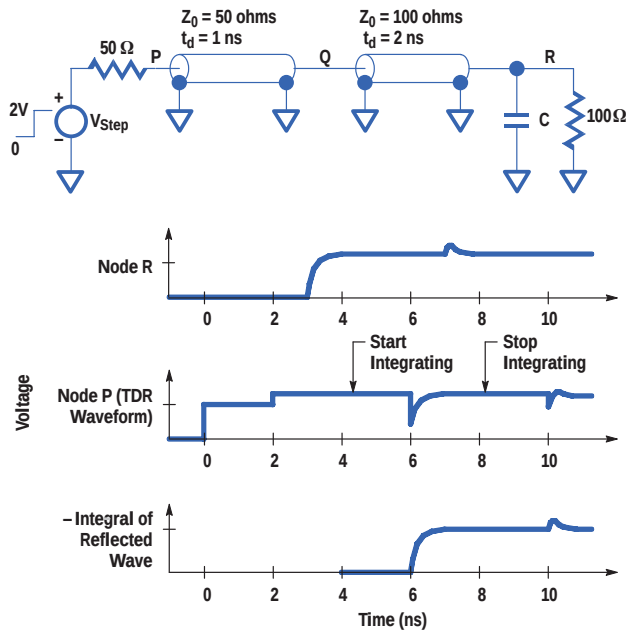
The length of the transmission line to the right of the discontinuity has no effect on the reflected or transmitted waveforms since it is terminated in its characteristic impedance. It has been shown as a long transmission line so the transmitted wave can be viewed easily. If the right transmission line is removed then the discontinuity is at the termination of the left transmission line. Quantifying parasitic inductance and capacitance at the termination of a transmission line is no different from quantifying parasitic inductance and capacitance along a transmission line.

Non-50-Ohm Z_{Ref}

Often, the impedance of the system to be measured is not the same as the impedance of the TDR/oscilloscope system. There are two ways to deal with non-50-ohm systems. First, the TDR can be connected directly to the system to be measured. Second, an impedance matching network can be inserted between the TDR and the system to be measured. In either case, the above relationships can be applied if corrections are made to the TDR waveform.

Connecting the TDR directly to a non-50-ohm system, as shown in Fig. 10, creates a system with two discontinuities, one being the transition between the 50-ohm TDR and the non-50-ohm system and the other being the discontinuity to be measured. The incident step will change amplitude at the non-50-ohm interface before arriving at the discontinuity to be measured. The reflected wave from the discontinuity creates another reflection when it arrives at the non-50-ohm- to-50-ohm interface. This secondary reflection propagates back into the non-50-ohm system and generates another reflection when it arrives at the discontinuity being measured. If twice the time delay of the transmission line between the non-50-ohm interface and the discontinuity being measured is larger than the settling time of the first reflected wave, then the integral of the normalized reflected wave can be evaluated before any secondary reflections arrive. Otherwise, all of the secondary reflections must settle before the integral can be evaluated. If the discontinuity is purely capacitive or inductive, then the rereflections will not change the final value to which the integral converges.

Fig. 10. *A TDR connected to a non-50-ohm transmission line with a capacitance C to be measured. There are multiple reflections. The discontinuity can be accurately quantified as long as the reflection to be measured can be isolated from other reflections on the TDR waveform.*



Two corrections need to be made to the TDR waveform. First, the height of the step that is incident on the discontinuity, inc_ht, is not the same as the height of the step that was generated by the TDR, tdr_ht. Second, the reflected waveform that is viewed by the oscilloscope, reflected_scope, has a different magnitude from the waveform that was reflected from the discontinuity, reflected. The relationships between the characteristic impedance of the system being measured, the characteristic impedance of the TDR, and the above waveforms are:

$$\text{inc_ht} = \text{tdr_ht}\frac{2Z_{Ref}}{Z_{Ref} + Z_{TDR}}$$

$$\text{reflected_scope} = \text{reflected}\frac{2Z_{TDR}}{Z_{TDR} + Z_{Ref}}.$$

The characteristic impedance of the system being measured can be calculated from the height of the step reflected from the transition between the 50-ohm TDR system and the non-50-ohm (Z_{Ref}) system, ref1_ht.

$$Z_{Ref} = Z_{TDR}\left(\frac{\text{incident} + \text{reflected}}{\text{incident} - \text{reflected}}\right)$$

$$= 50\ \text{ohms}\left(\frac{\text{tdr_ht} + \text{ref1_ht}}{\text{tdr_ht} - \text{ref1_ht}}\right).$$

To calculate series inductance or shunt capacitance, the integral of the waveform reflected from the discontinuity, normalized to the height of the step incident on the discontinuity, needs to be evaluated. In terms of the measured TDR waveform, the normalized reflected waveform is:

$$\text{reflected_n} = \frac{\text{reflected}}{\text{inc_ht}} = \frac{\text{reflected_scope}\left(\dfrac{Z_{Ref} + Z_{TDR}}{2Z_{TDR}}\right)}{\text{tdr_ht}\left(\dfrac{2Z_{Ref}}{Z_{Ref} + Z_{TDR}}\right)}$$

$$= \frac{\text{reflected_scope}}{\text{tdr_ht}}\frac{\left(Z_{Ref} + Z_{TDR}\right)^2}{4Z_{Ref}Z_{TDR}}.$$

As an example, the shunt capacitance would be:

$$C = -\frac{2}{Z_{Ref}} \int_0^{+\infty} \text{reflected_n} \cdot dt$$

$$= -\frac{\left(Z_{Ref} + Z_{TDR}\right)^2}{2Z_{TDR}Z_{Ref}^2} \int_0^{+\infty} \frac{\text{reflected_scope}}{\text{tdr_ht}} dt.$$

Reflections between the discontinuity and the non-50-ohm transition can be removed if an impedance matching network is inserted between the TDR and the non-50-ohm system. Two resistors are the minimum requirement to match an impedance transition in both directions. Attenuation across the matching network needs to be accounted for before integrating the reflected waveform. When a matching network is used, neither the impedance of the system being tested nor the attenuation of the matching network can be ascertained from the TDR waveform. If tran_12 is the gain of the matching network going from the TDR to the system and tran_21 is the gain going from the system to the TDR, then the normalized reflected waveform is:

$$\text{reflected_n} = \frac{\text{reflected}}{\text{inc_ht}} = \frac{\left(\frac{\text{reflected_scope}}{\text{tran_21}}\right)}{\text{tdr_ht} \cdot \text{tran_12}}$$

$$= \frac{\text{reflected_scope}}{\text{tdr_ht}} \frac{1}{\text{tran_12} \cdot \text{tran_21}}.$$

Series Capacitance

So far, all of the discontinuities that have been measured have been mere perturbations in the characteristic impedance of transmission line systems. The discontinuities have been quantified by analyzing the reflected waveforms using TDR. Now consider a situation where a capacitance to be measured is in series with two transmission lines, as shown in Fig. 11. The series capacitance can be quantified using the transmitted waveform rather than the reflected waveform, or using *time-domain transmission* (TDT). Given an incident step that arrives at the series capacitance at time = 0, the transmitted waveform is zero for time < 0. For time > 0, the transmitted waveform is:

$$\text{transmitted} = \text{inc_ht} \cdot e^{-\frac{t}{\tau}},$$

where $\tau = RC = 2Z_0C.$

Note that τ is now proportional to $2Z_0$ instead of $Z_0/2$. Normalizing the transmitted waveform to the incident step height and integrating the normalized, transmitted waveform gives:

$$\text{transmitted_n} = \frac{\text{transmitted}}{\text{inc_ht}}$$

$$\int_{-\infty}^{+\infty} \text{transmitted_n} \cdot dt = \int_0^{+\infty} e^{-\frac{t}{\tau}} \cdot dt = -\tau e^{-\frac{t}{\tau}} \Big|_0^{+\infty} = \tau.$$

Solving for the series capacitance gives:

$$C = \frac{\tau}{2Z_0} = \frac{1}{2Z_0} \int_0^{+\infty} \text{transmitted_n} \cdot dt.$$

Shunt Inductance

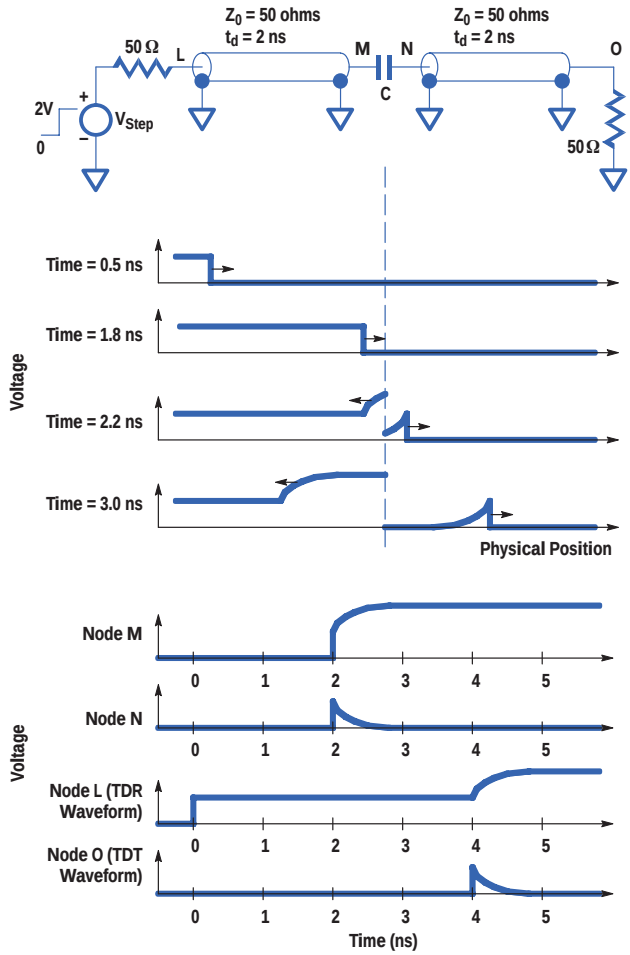
The last topology to be analyzed will be shunt inductance to ground. Given an incident step that arrives at the shunt inductance at time = 0, the transmitted waveform is zero for time < 0. For time > 0, the transmitted waveform is:

$$\text{transmitted} = \text{inc_ht} \cdot e^{-\frac{t}{\tau}},$$

where $\tau = \frac{L}{R} = \frac{L}{\left(Z_0/2\right)} = \frac{2L}{Z_0}.$

Fig. 11. A series capacitance between two transmission lines can be measured using the transmitted waveform (time-domain transmission or TDT) rather than TDR.

Click to View Animated Figure 11



This is the same transmitted waveform as in the case of series capacitance. τ is $2L/Z_0$, so the shunt inductance is:

$$L = \frac{Z_0}{2}\tau = \frac{Z_0}{2}\int_0^{+\infty} \text{transmitted_n} \cdot dt.$$

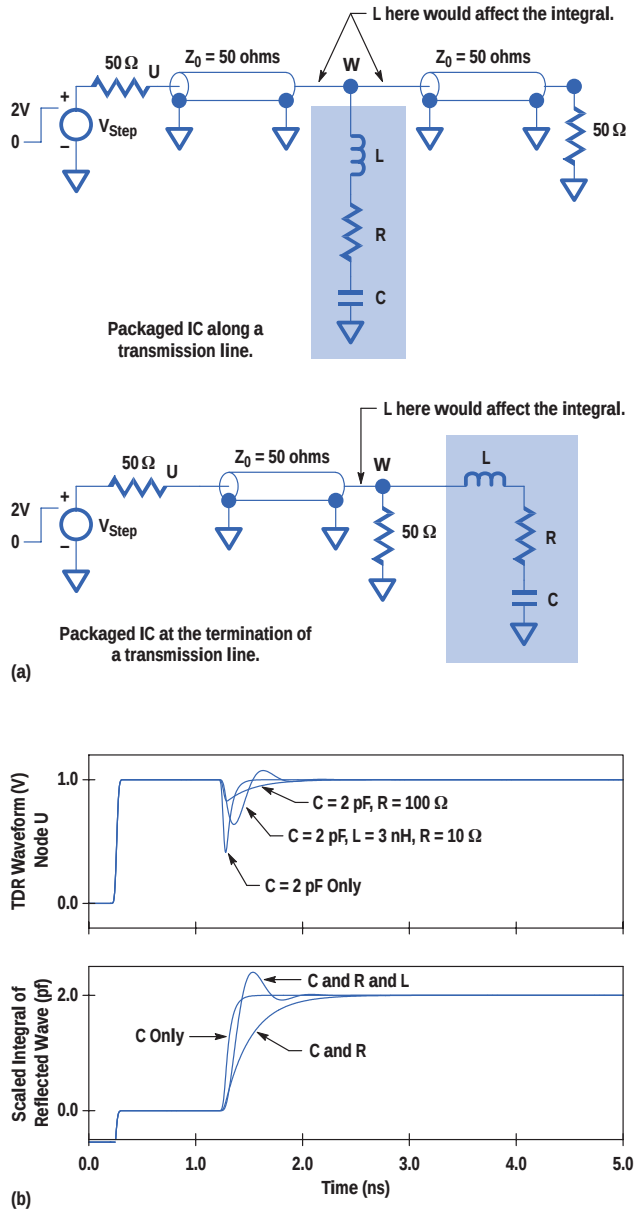
More Complex Discontinuities

Consider a circuit in which a transmission line is brought to the input of an integrated circuit (IC) and terminated (Fig. 12). Looking from the termination into the IC, there might be inductance of the lead in series with a damping resistance in series with the input capacitance of the IC. A first-order approximation of the load that the IC puts on the termination of the transmission line is simply a capacitor. The reflected wave can be integrated to calculate the capacitance, but will the series inductance and resistance change the measured value? Looking back to Fig. 5, the current into the shunt capacitor is:

$$I = \frac{2\text{inc_ht}}{Z_0} e^{-\frac{t}{\tau}} = -\text{reflected}\frac{2}{Z_0}.$$

The integral of the reflected wave is directly proportional to the integral of the current into the capacitor. This means that the value of capacitance calculated from integrating the reflected wave is proportional to the total charge on the capacitor. Resistance or inductance in series with the shunt capacitor does not change the final value of charge on the capacitor. Hence, these elements do not change the measured value of capacitance. Series resistance decreases the height and increases the settling time of the reflected wave. Series inductance superimposes ringing onto the reflected waveform. Neither changes the final value of the integral of the reflected wave. Note that inductance in series with the transmission line, as opposed to inductance in series with the capacitor, will change the measured value of capacitance. Similarly, when measuring series inductance, resistance or capacitance in parallel with the series inductance does not change the measured value of inductance.

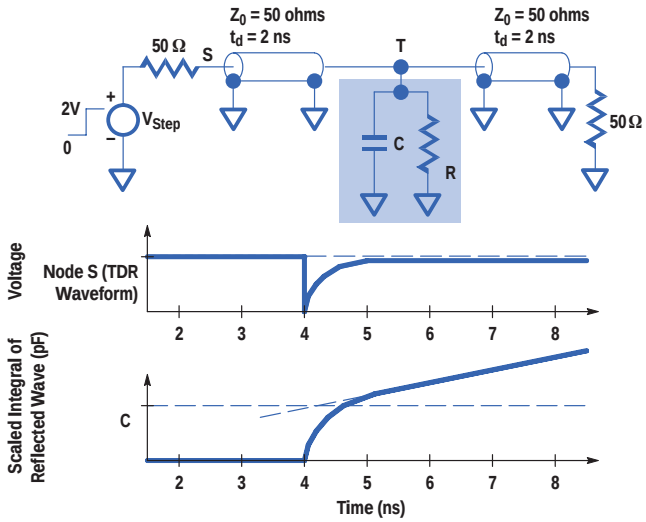
Fig. 12. L and R in series with a shunt capacitance do not affect the measurement of C.



When building a model for the input of the packaged IC, integrating the reflected wave provides the correct value of capacitance for a single capacitor model and also for higher-order models. If, for example, a higher-order model contains a section of transmission line terminated with some capacitive load, then the value of capacitance measured is the total capacitance of the transmission line plus the load. To ascertain values for L and R in Fig. 12, a more rigorous analysis of the TDR waveform must be performed.

Now consider a situation in which a voltage probe is used to measure signals on printed circuit board traces. What effect does the probe have on signals that are propagating along the traces? To a first-order approximation, the load of a high-impedance probe will be capacitive and the value of capacitance can be measured by integrating the reflected wave. However, some passive probes that are optimized for bandwidth can also have a significant resistive load (these probes usually have a much lower capacitive load than high-impedance probes). Looking at the reflected wave in Fig. 13, the resistive load of the probe introduces a level shift at 4 ns. The level shift adds a ramp to the integral so that the integral never converges to a final value. To measure the value of capacitance, the integral can be evaluated at two different times after the reflected wave has settled to its final value, say at 6 and 7 ns. Since the reflected wave has settled, the difference between the two values is caused only by the level shift. The component of the integral resulting from the level shift can be removed from the total integral by subtracting twice the difference between the values at 6 and 7 ns from the value at 6 ns. The remaining component of the integral is a result of the shunt capacitance.

Fig. 13. A voltage probe used to measure signals on printed circuit board traces may act like a shunt RC load, adding a ramp to the integral of the reflected wave.



Multiple Discontinuities

Sometimes, the transmission lines between the TDR/oscilloscope and the discontinuity to be measured are less than ideal. For example, the connection between a 50-ohm coaxial cable and a 50-ohm printed circuit board trace can look quite inductive if a short jumper wire is used. Also, the system being measured may have more than one discontinuity.

Intermediate discontinuities between the TDR and the discontinuity to be measured act as low-pass filters that increase the rise times of both the incident step and the reflected wave. Intermediate discontinuities also cause secondary reflections in the TDR waveform, as shown in Fig. 14. If the discontinuity to be measured can be separated from surrounding discontinuities on the TDR waveform, then the discontinuity can be accurately quantified by integrating the reflected wave. Increasing the rise time of the system decreases any separation between discontinuities but, as was shown earlier, does not change the integral of the reflected wave.

Minimizing Effects of Discontinuities (Balancing)

There are many instances in which discontinuities in transmission line systems are unavoidable. The effects of an unavoidable discontinuity can be minimized by introducing discontinuities of opposite polarity as close as possible to the unavoidable discontinuity. For example, if the narrow part of the trace shown in Fig. 8 is required, then the traces on either side of the narrow trace can be made wider to compensate for the excess inductance of the trace. The amount of excess capacitance required to compensate for the 4.88 nH of excess inductance is $Z_0^2L = (50\text{ ohms})^2 \times 4.88$ nH = 1.95 pF. If the widest trace that can be added has a characteristic impedance of 30 ohms, then 152 ps of this trace will add 1.95 pF of excess capacitance. Propagation delays on typical printed circuit board traces are around 150 ps/inch, so adding 0.5 inch of 30-ohm transmission line on each side of the narrow trace will balance the excess inductance of the trace. Balancing the discontinuity reduces reflections and makes the transmitted wave more closely

Fig. 14. Intermediate discontinuities do not affect the measurement as long as the discontinuity to be measured can be isolated on the TDR waveform.

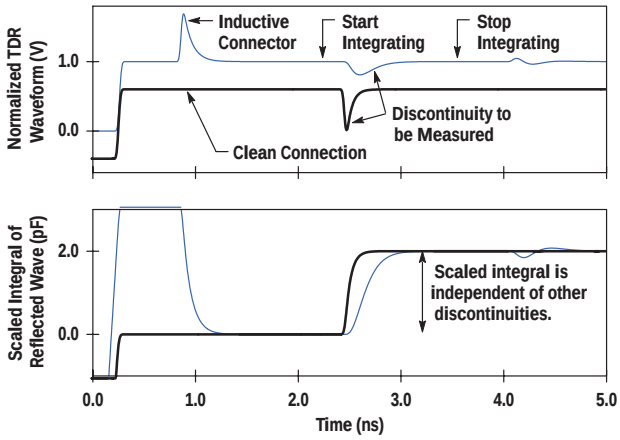


Fig. 15. Measuring the input capacitance of an HP 54701A active probe by placing cursors around the reflection caused by the probe.

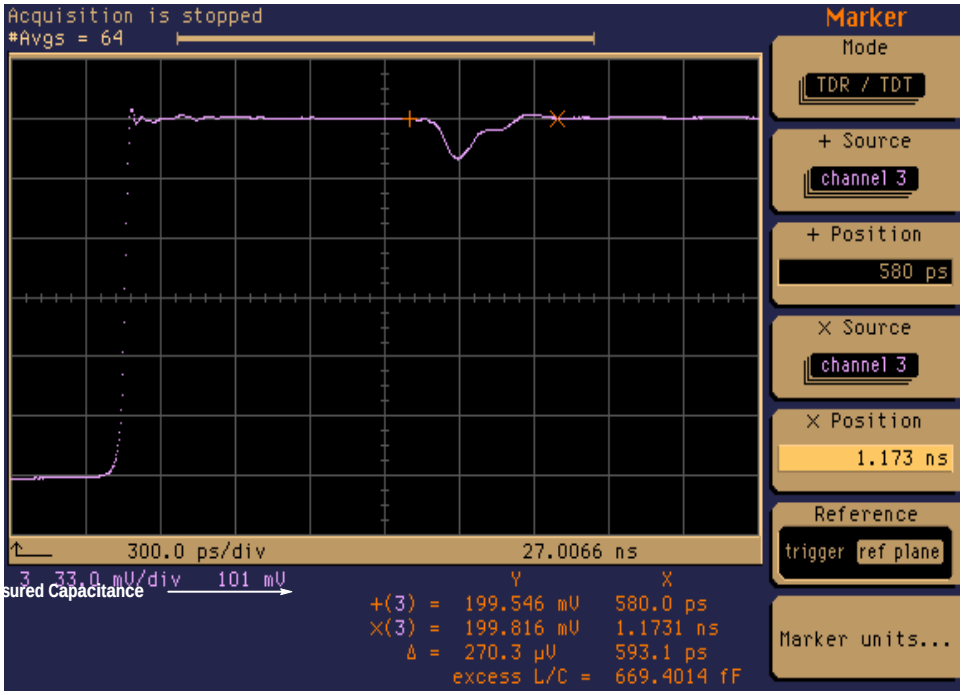
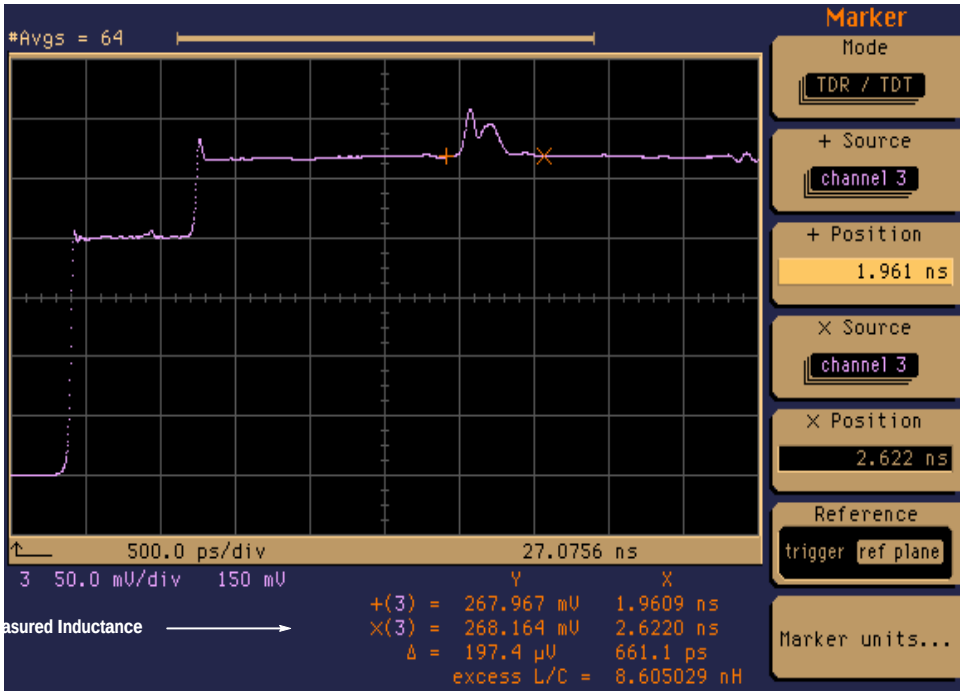


Fig. 16. Measuring the excess inductance of a connector in a 100-ohm system by placing cursors around the reflection caused by the inductance.



resemble the incident wave. A discontinuity that is balanced will produce a reflected wave whose integral is zero. In this case, the integral of the reflected wave can be used to determine how well a discontinuity is balanced.

Instrumentation

Hewlett-Packard recently introduced the HP 54750A oscilloscope and the HP 54754A TDR plug-in. All of the measurements discussed above can be made by positioning two cursors on the TDR or TDT waveform. For TDR, the position of the leftmost cursor defines when to start integrating the reflected wave and also determines the reference impedance for the calculation of excess L or C. The position of the rightmost cursor determines when

to stop integrating the reflected wave. Likewise, when using TDT, the cursors determine when to start and stop integrating the transmitted wave. The oscilloscope calculates the integral of the normalized reflected or transmitted wave and displays the appropriate value of capacitance or inductance, as shown in Figs. 15 and 16. Parasitic inductances and capacitances can be accurately measured in the time it takes to position two cursors.

Summary

TDR can be used to measure impedances and discontinuities in transmission line systems very accurately. Discontinuities that are either primarily capacitive or primarily inductive can be accurately modeled by a single element whose value can be determined by integrating the reflected or transmitted waves. In 50-ohm systems, the integral of the normalized reflected wave is simply the time constant of the network.

Discontinuities can also be quantified in terms of the peak value of the reflections they generate. The problem with using the peak value of the reflection is that the value depends on the rise time of the incident step and the loss of the transmission line system between the TDR and the discontinuity. Using the integral of the reflected wave to quantify a discontinuity makes the measurement independent of the rise time of the incident step and also independent of discontinuities between the TDR and the discontinuity being measured, as long as the discontinuity being measured can be isolated from surrounding discontinuities on the TDR waveform.

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